

# Two-way perturbative coupling of plasma-fluid interactions

GEC 2025

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Supervisors: D. Fabre, F. Plouraboué

IPROP EIC Pathfinder : <https://www.iprop-project.eu/>

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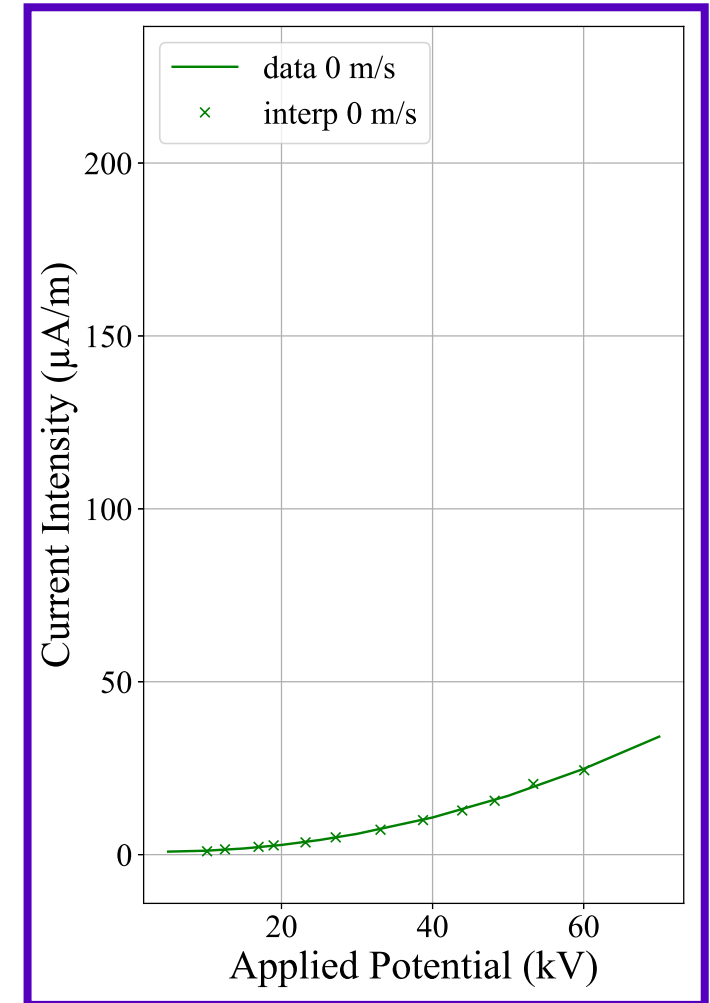
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# Current Intensity of DC Corona Discharge in a Flow

- Chapman's measurements in the 60's

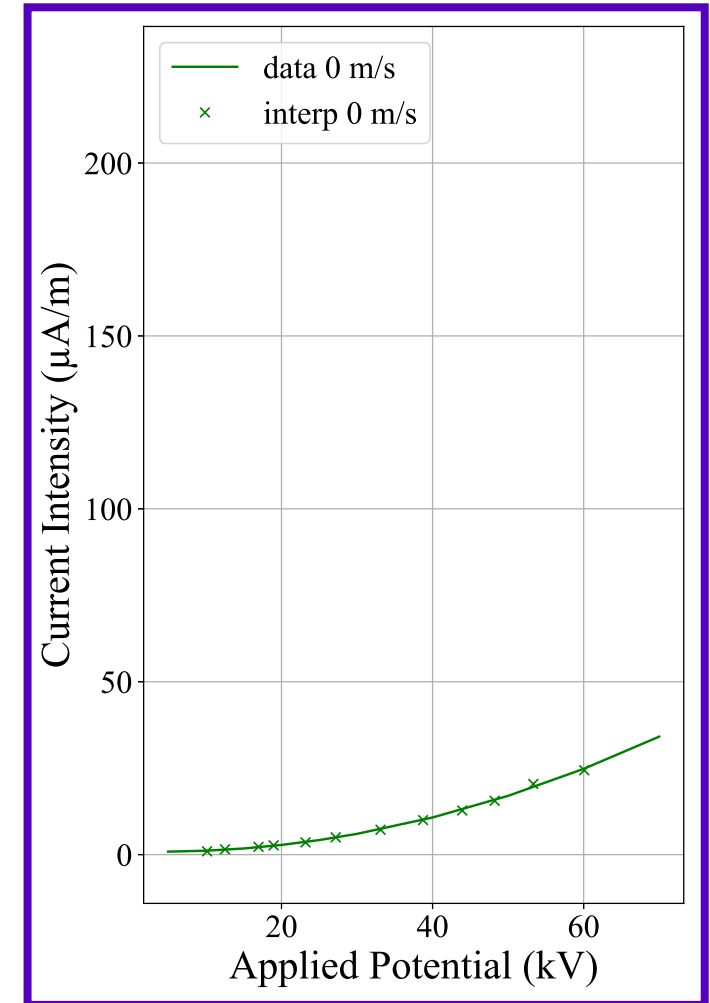


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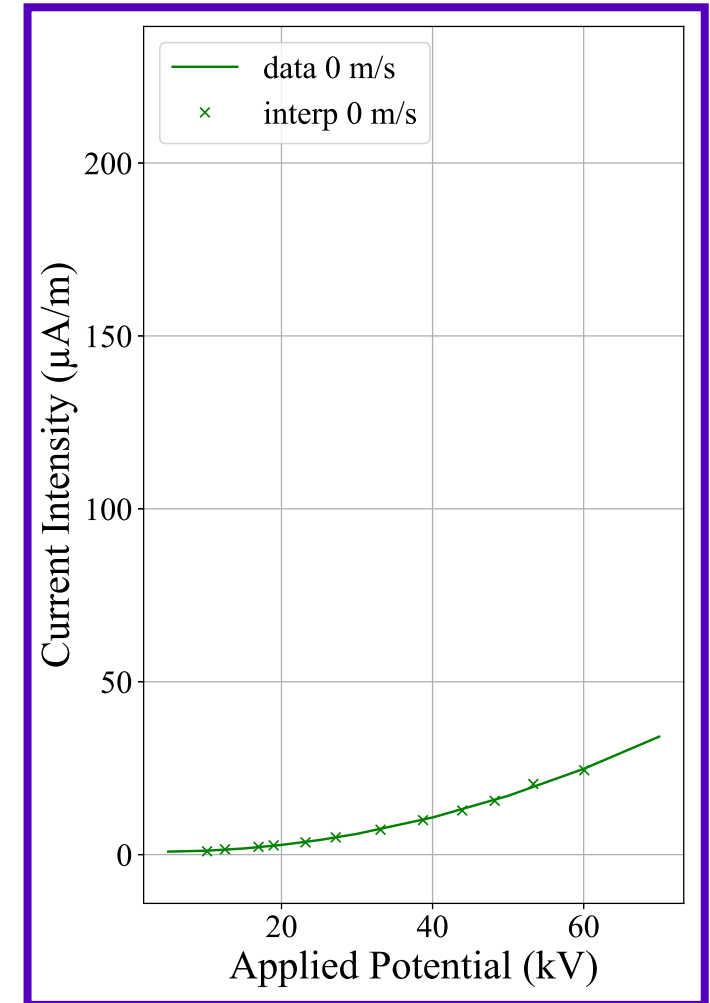
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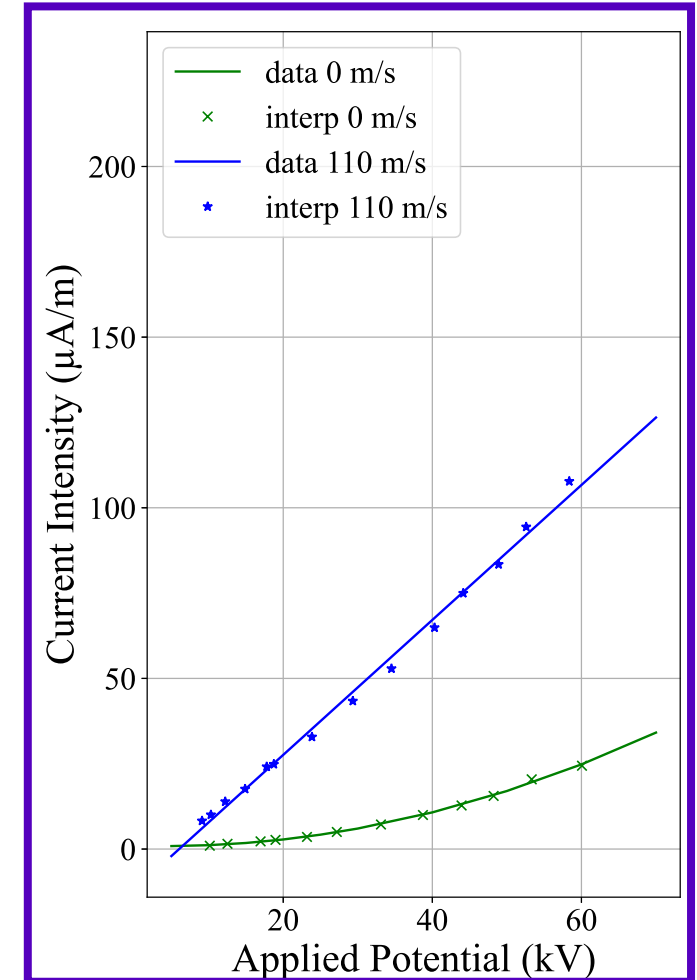
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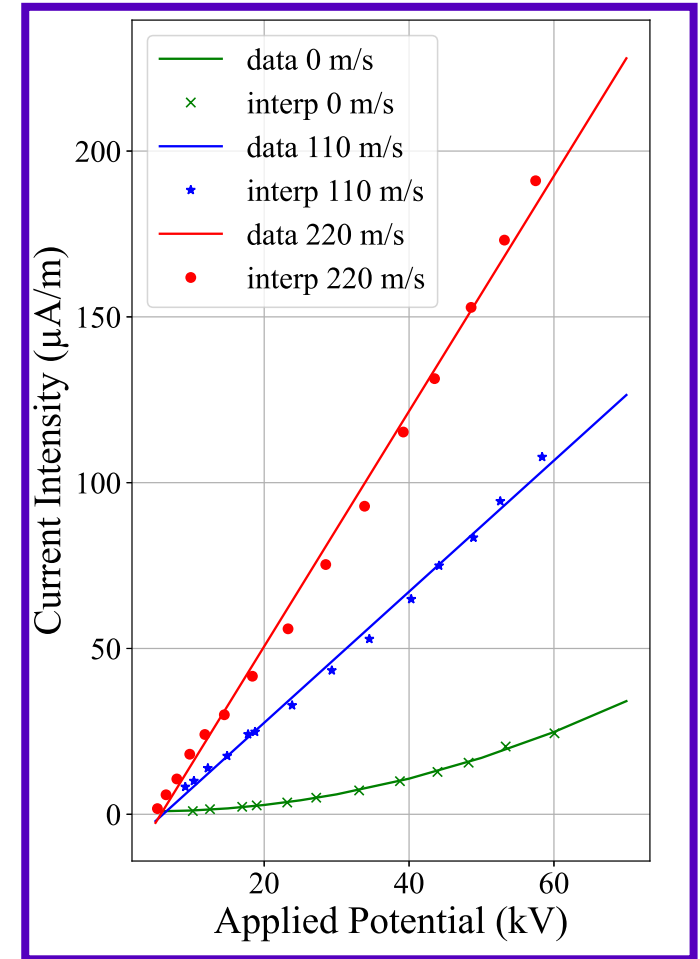
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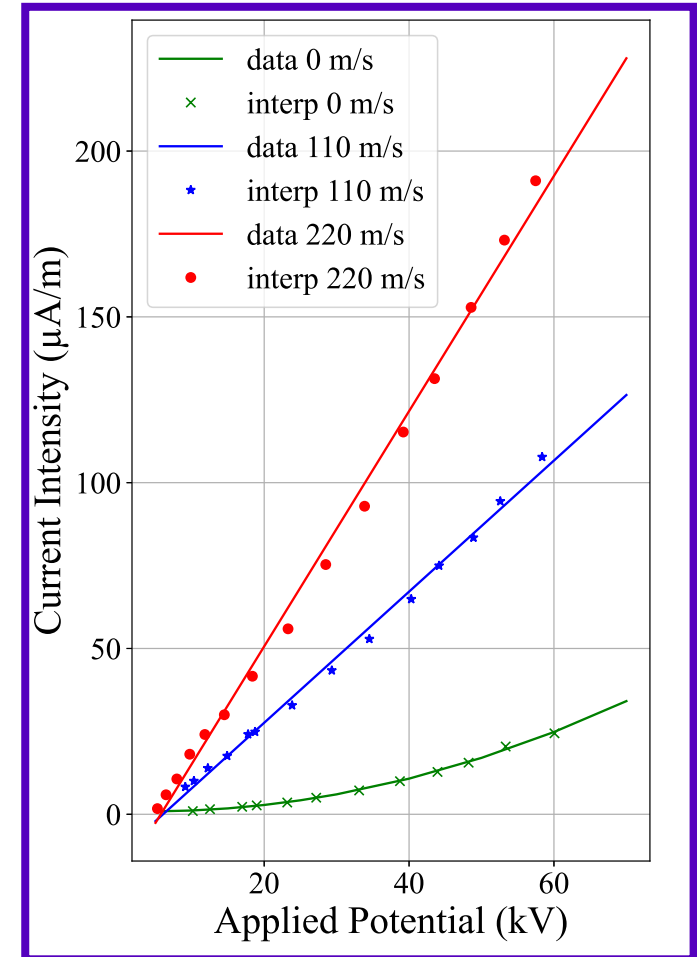
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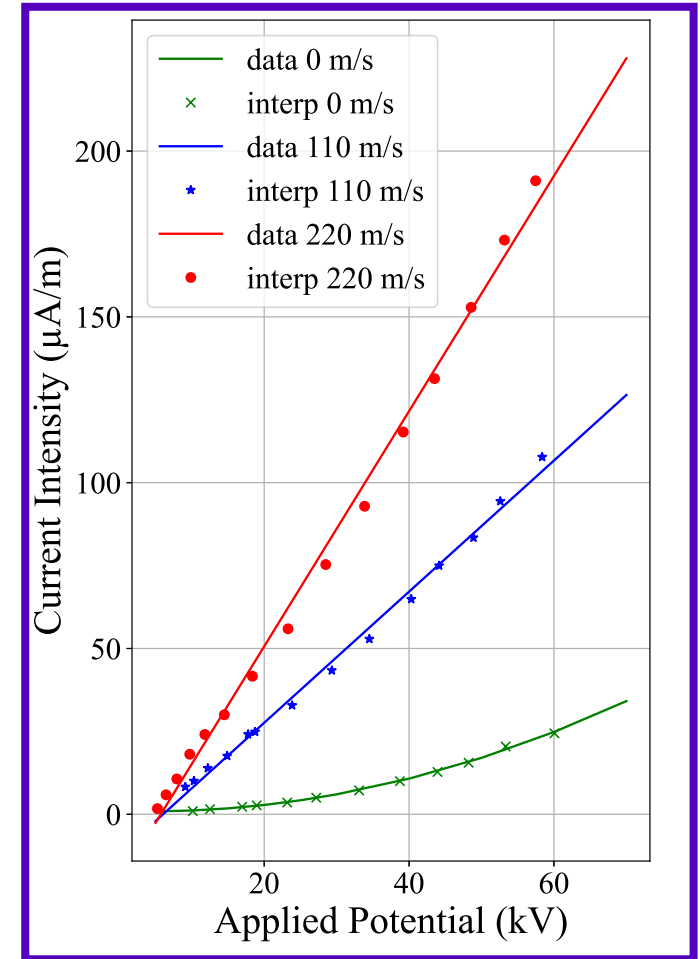
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⇒ Modelling ?



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# Dimensionless Two-Way Model of Plasma & Fluid

## Physical Parameters

Electro-drift Péclet number :

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Monrolin & Plouraboué. *J. Comp, Phys.*, 443:110517 (2021).

Picella et al. *AIAA J.*, 62-7 pp1–12, (2024).

[josemaria.diascoelho@toulouse-inp.fr](mailto:josemaria.diascoelho@toulouse-inp.fr)

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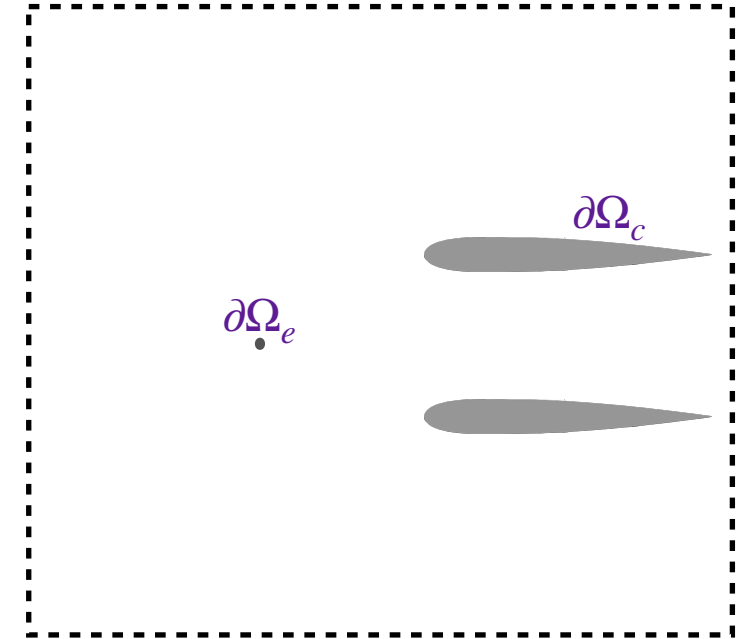
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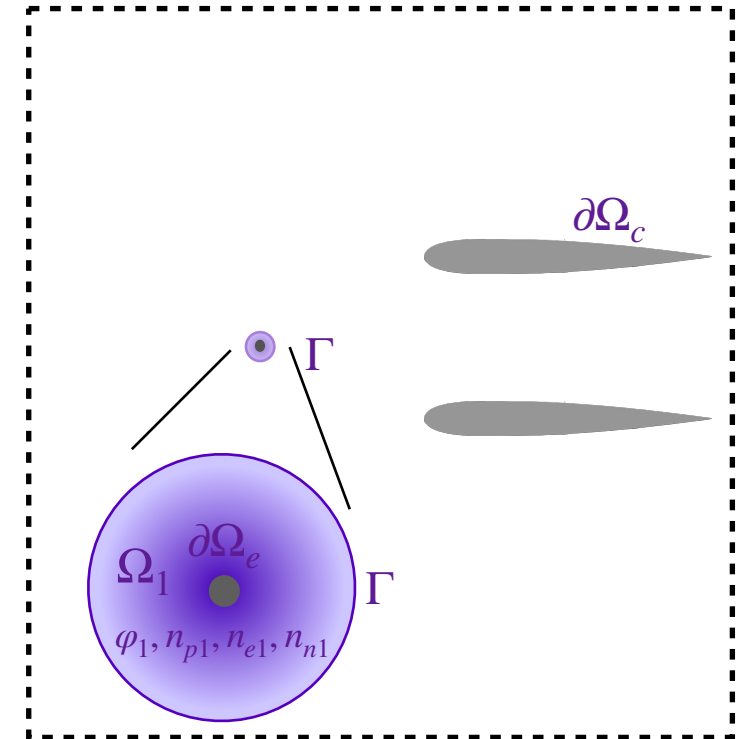
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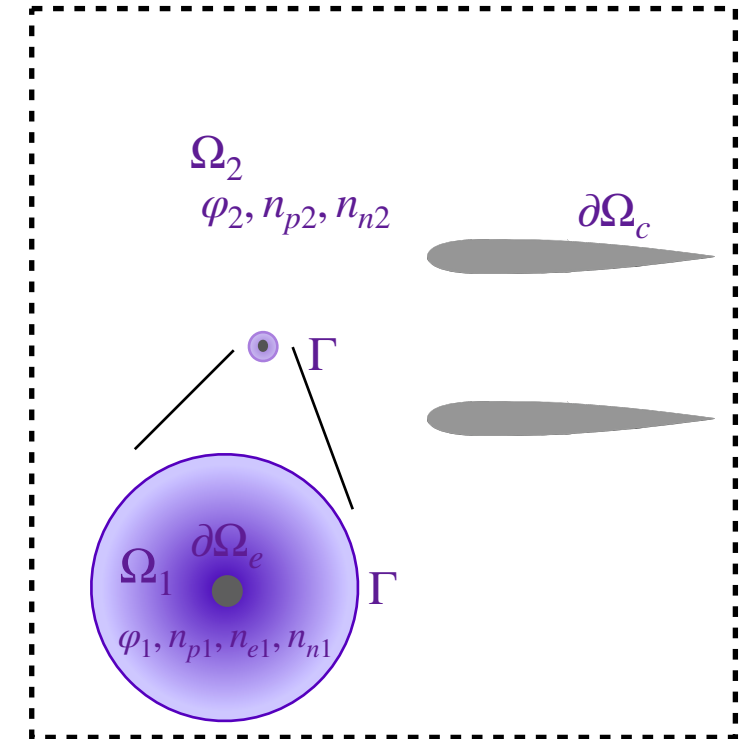
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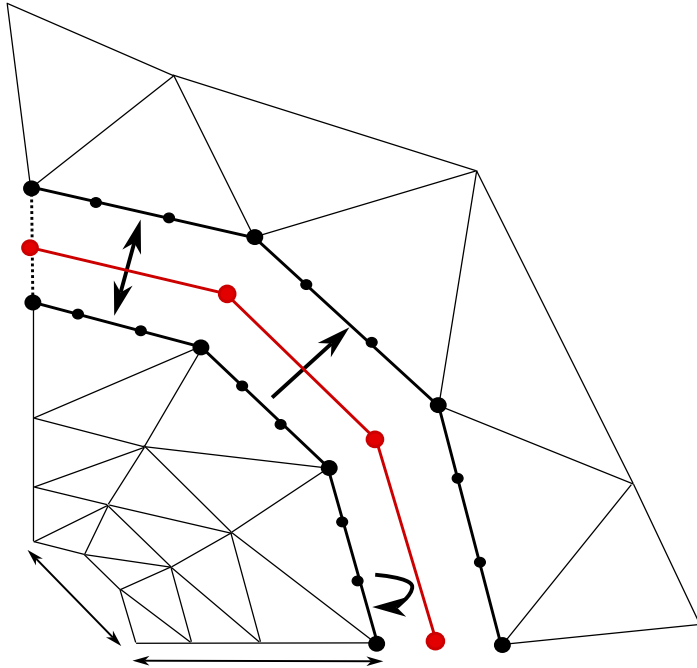
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# Two-Way Model (Some) Numerical Aspects



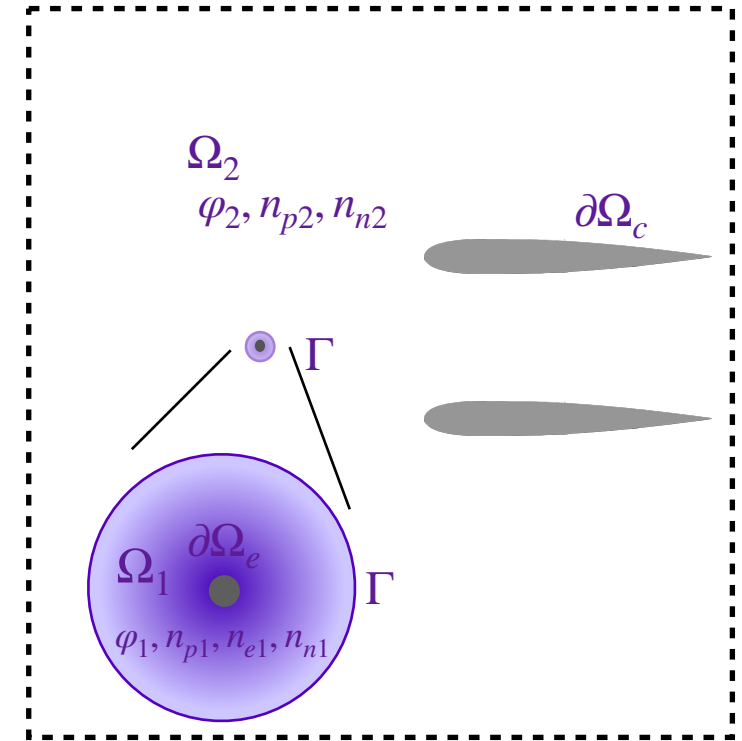
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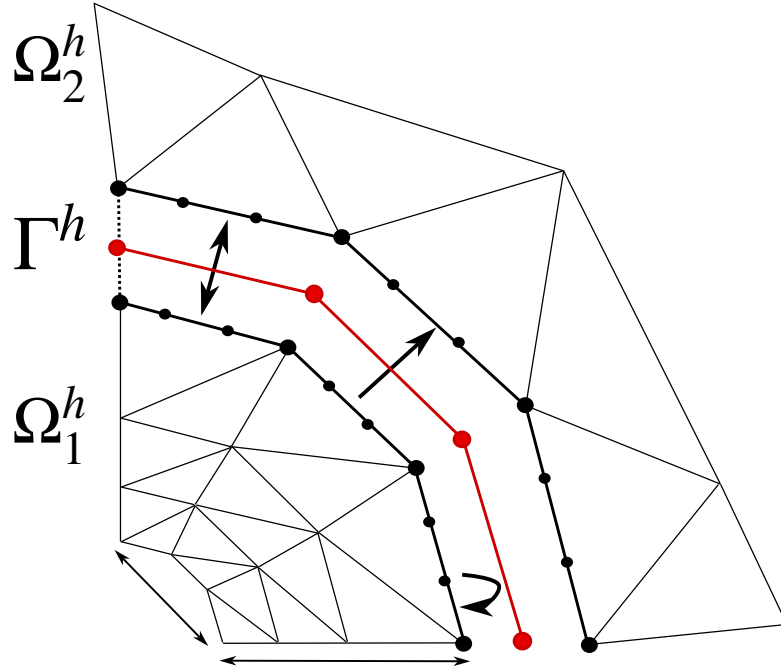
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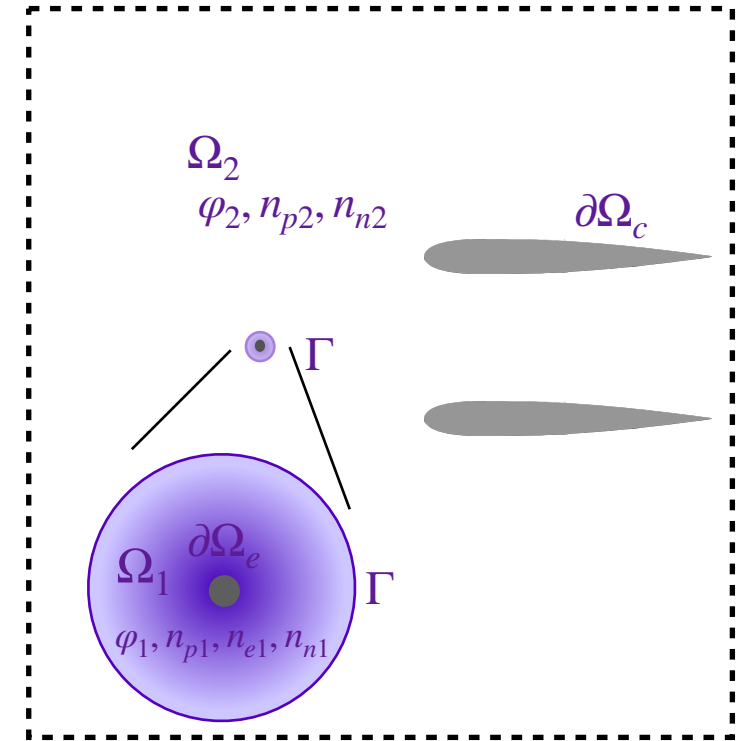
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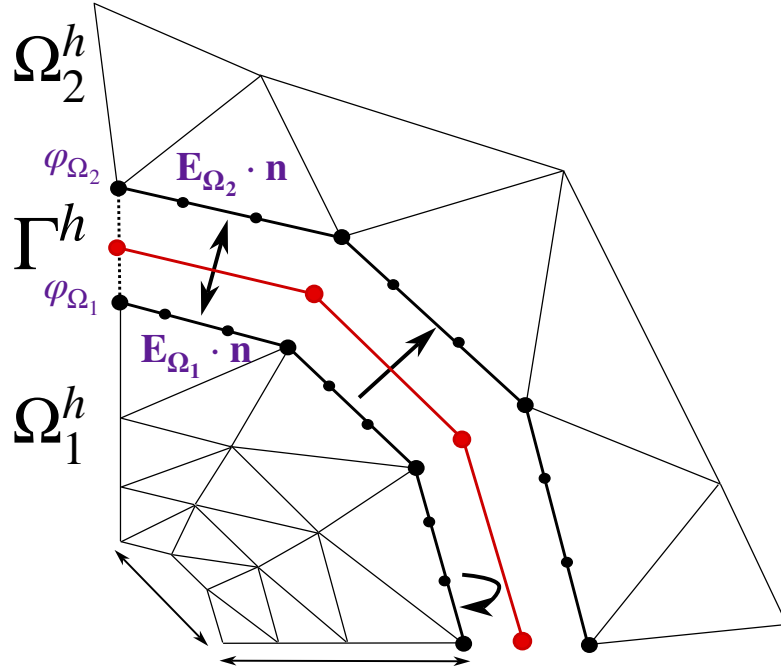
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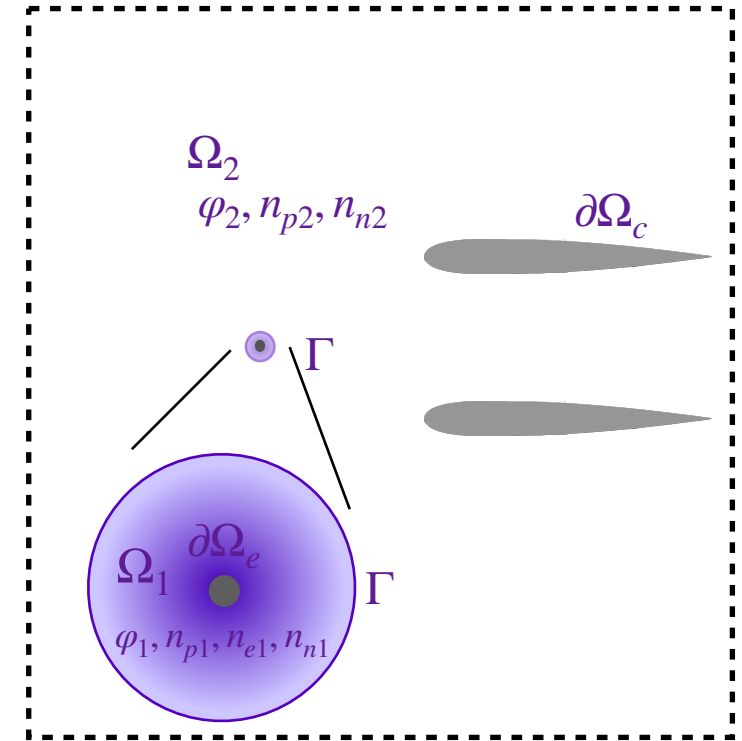


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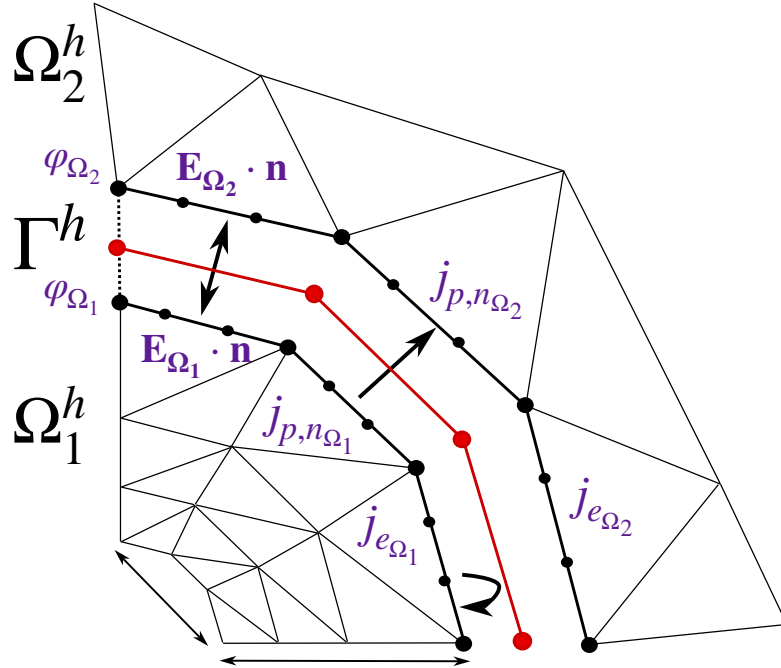
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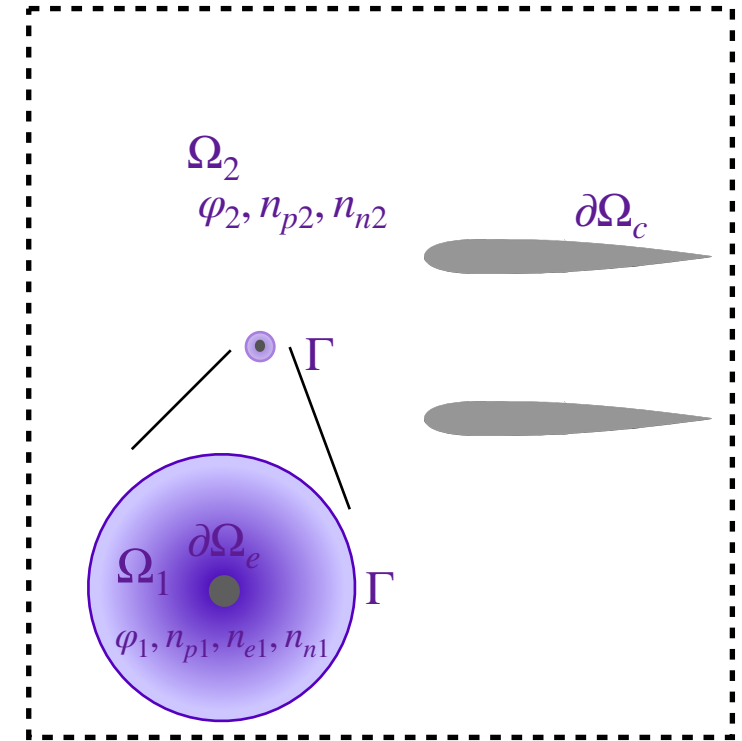


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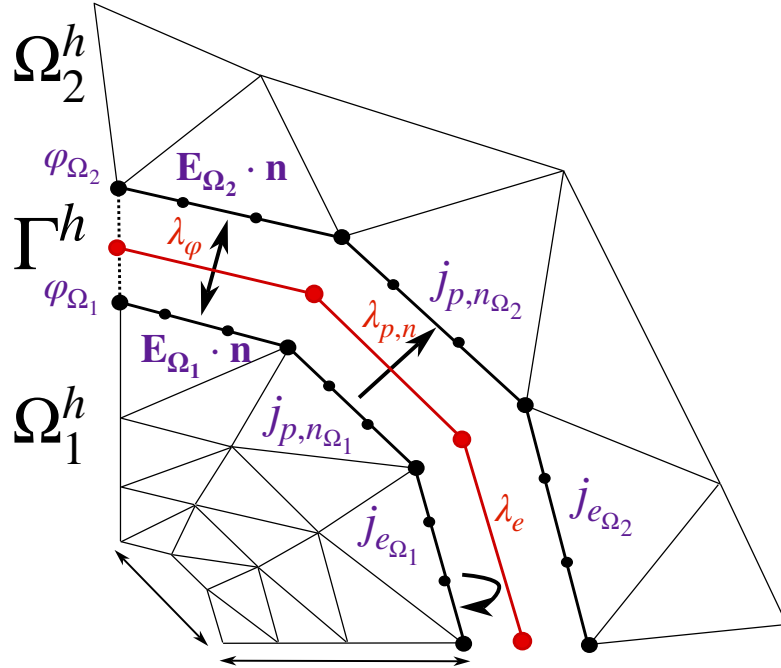
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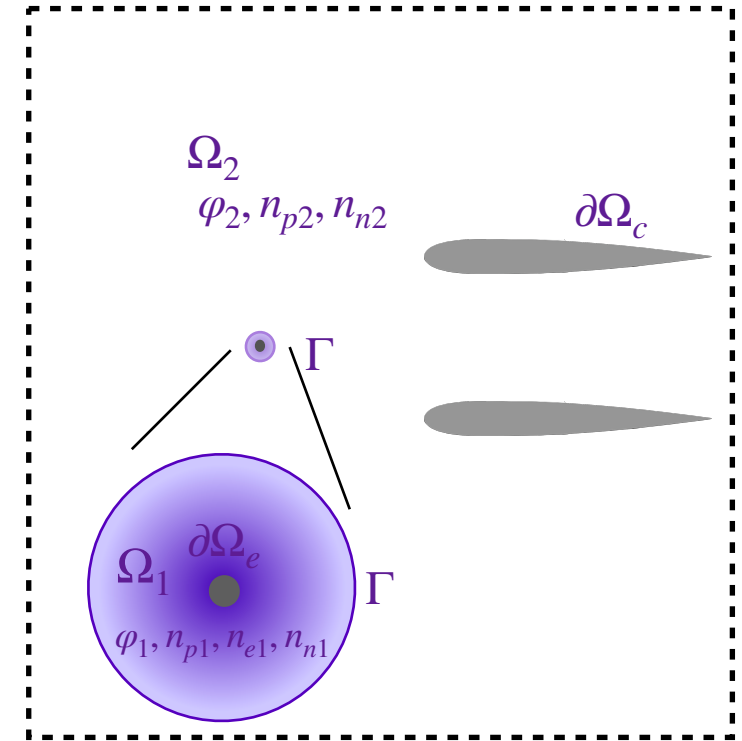


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C. Two-way coupling : charge  
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$$\text{EHD Macro-to-Micro speed ratio : } M_m = \frac{U_e}{\mu E} = \frac{1}{\mu_p} \sqrt{\frac{\epsilon_0}{\rho_f^*}} \approx 10^{-2}$$



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$$U_e = \frac{\varphi_a}{r_c} \sqrt{\frac{\epsilon_0}{\rho_f}}$$

Speed ratio :

$$\beta = \frac{U_{in}}{U_e}$$

Inner domain  $\Omega_1$

$$\begin{cases} \Delta \varphi = 0 \\ \nabla \cdot [n_p(-\nabla \varphi + M_m \mathbf{u})] = \alpha \|n_e \mathbf{E}\| \\ \nabla \cdot [n_e(\nabla \varphi + M_m \mathbf{u})] = (\alpha - \eta) \|n_e \mathbf{E}\| \\ \nabla \cdot [n_n(\nabla \varphi + M_m \mathbf{u})] = \eta \|n_e \mathbf{E}\| \end{cases}$$

Outer domain  $\Omega_2$

$$\begin{cases} \Delta \varphi = -n_p + n_n \\ \nabla \cdot [n_p(-\nabla \varphi + M_m \mathbf{u}) - \frac{1}{P_e} \nabla n_p] = \gamma S \\ \nabla \cdot [n_n(\nabla \varphi + M_m \mathbf{u}) - \frac{1}{P_e} \nabla n_n] = 0 \end{cases}$$

Full domain  $\Omega$

$$\begin{cases} \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{R_e} \Delta \mathbf{u} - (n_p - n_n + n_e) \nabla \varphi \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

Asymptotic expansion with respect to :  $M_m = \frac{U_e}{\mu E} = \frac{1}{\mu_p} \sqrt{\frac{\epsilon_0}{\rho_f^*}} \approx 10^{-2}$

A. Corona Discharge



B. One-way coupling :  
Coulomb forcing

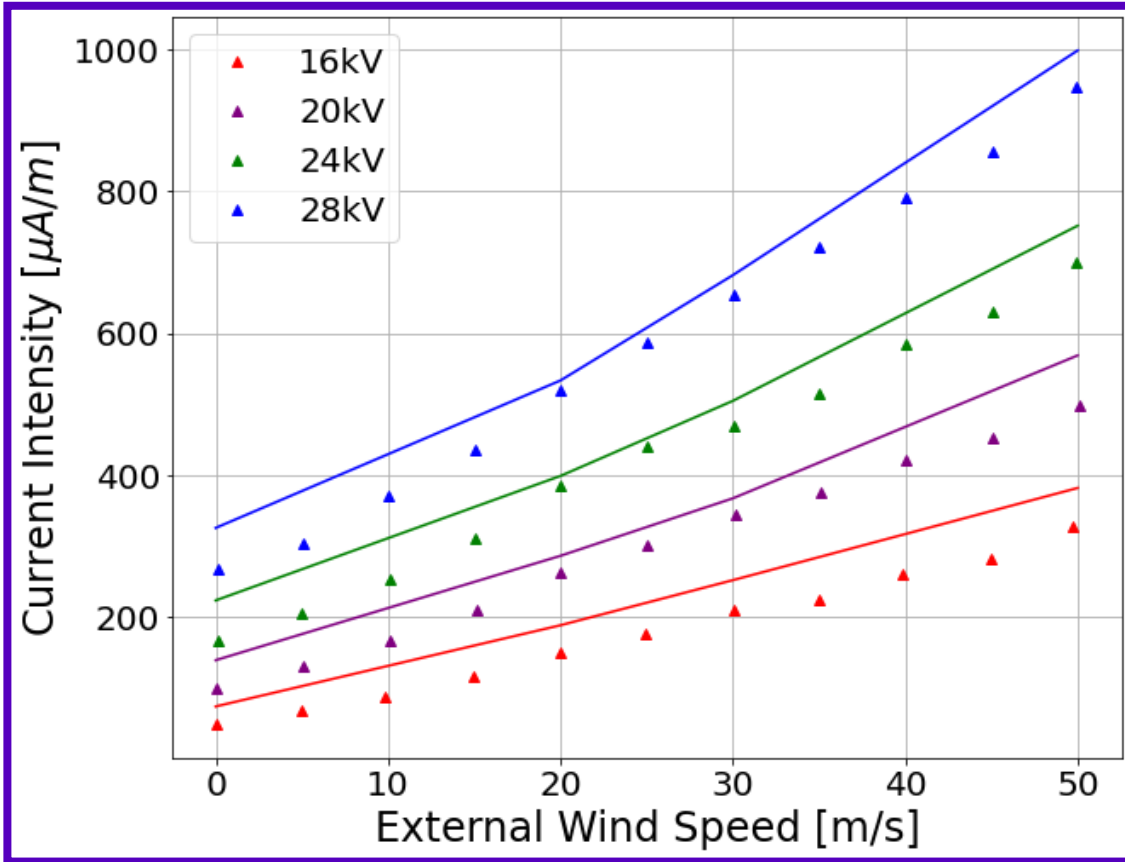


C. Two-way coupling :  
**Perturbative** retroaction  
of charge advection by the  
fluid onto the discharge



# Comparison with Experiments

## Cylindrical collector



Implemented with :

- FreeFem++
- StabFem++

Grosse et al., *J. Elect*, **130**-103950, 2025  
 Chapman. *J. Geo. Res.*, 75(12):**2165**–2169, 1970.

[josemaria.diascoelhomarques@toulouse-inp.fr](mailto:josemaria.diascoelhomarques@toulouse-inp.fr)

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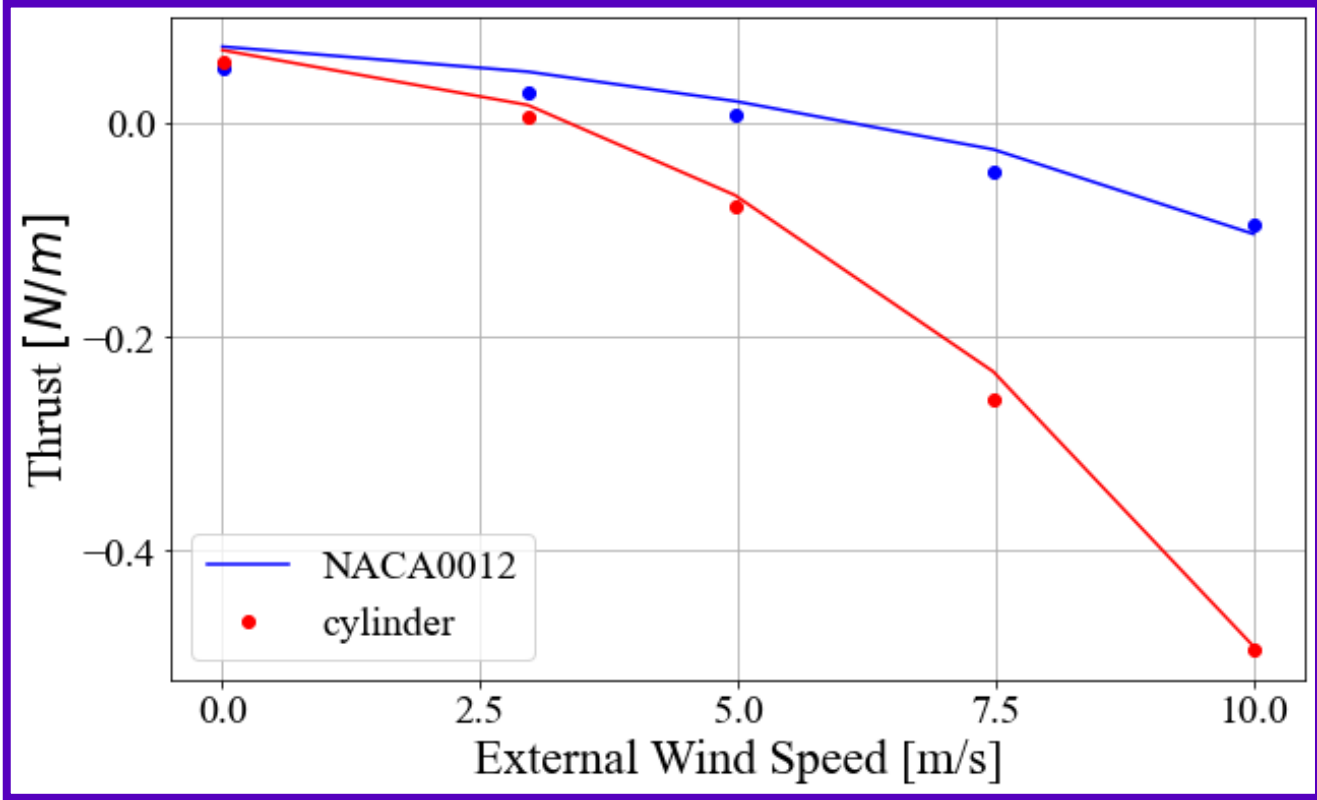
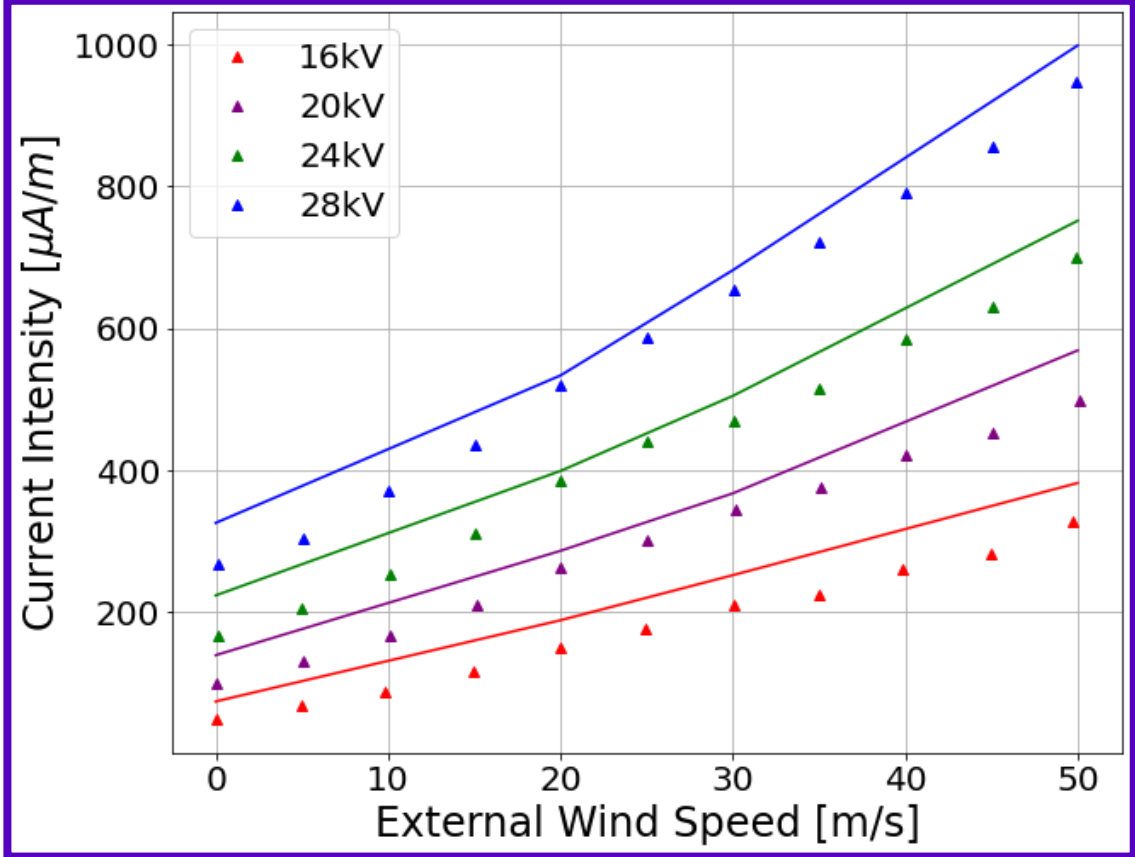
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the European Union

# Comparison with Experiments

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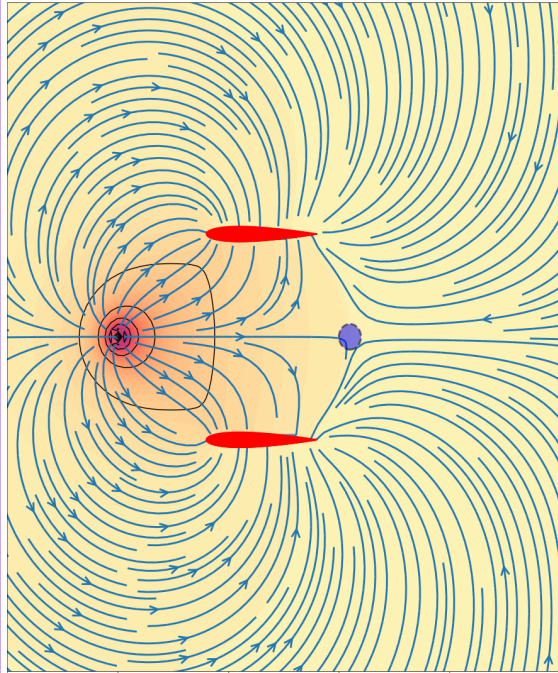
[josemaria.diascoelho@toulouse-inp.fr](mailto:josemaria.diascoelho@toulouse-inp.fr)

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# Electric Field Lines

$$U_{in} = 1 \text{ m/s}$$



$$U_{in} = 2.87 \text{ m/s}$$

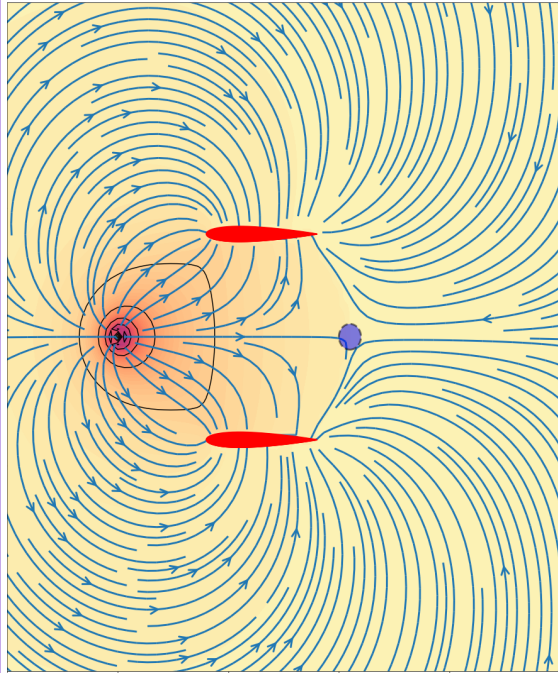
$$U_{in} = 5.73 \text{ m/s}$$

$$U_{in} = 12.26 \text{ m/s}$$

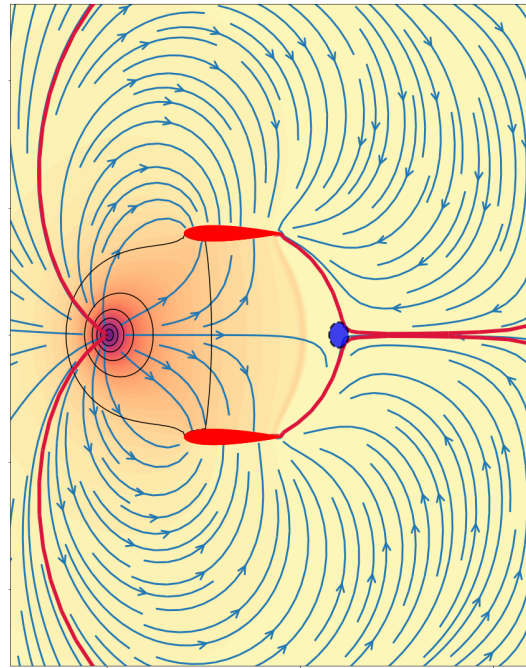


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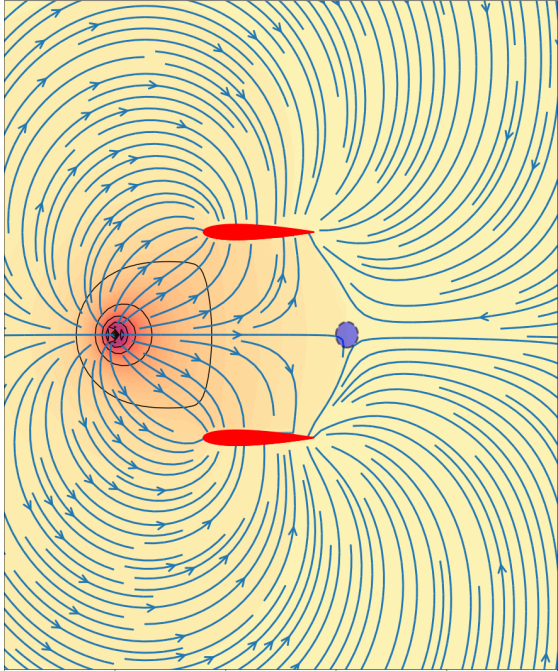


$U_{in} = 12.26 \text{ m/s}$

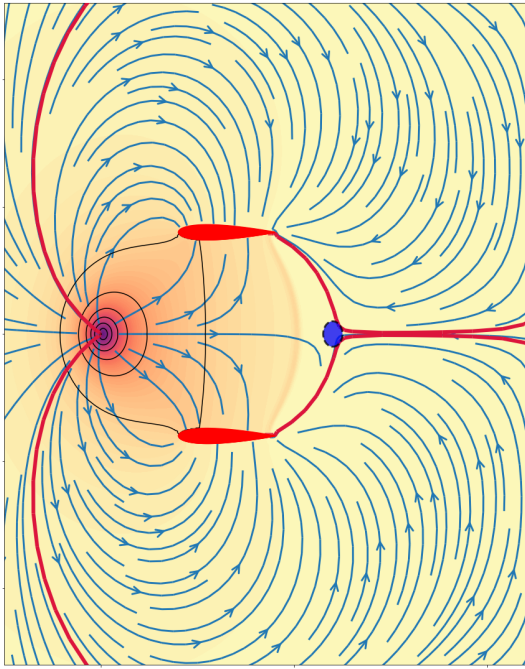


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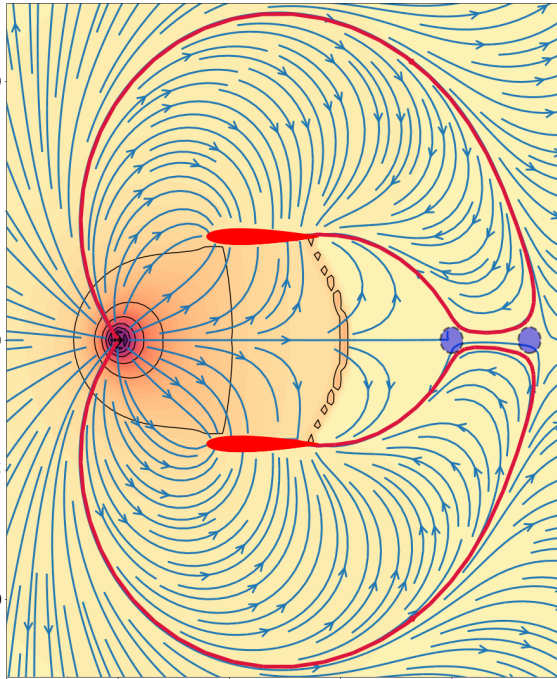
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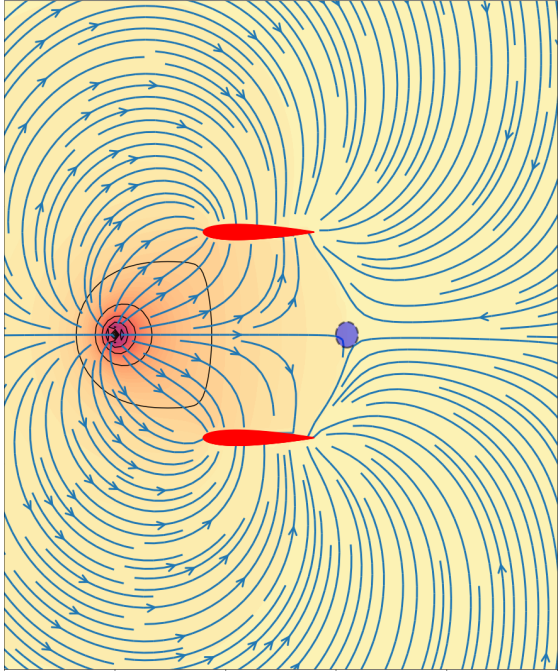


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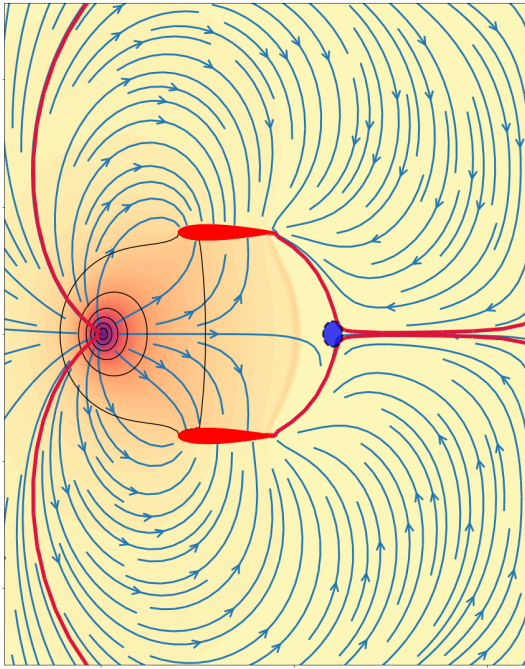


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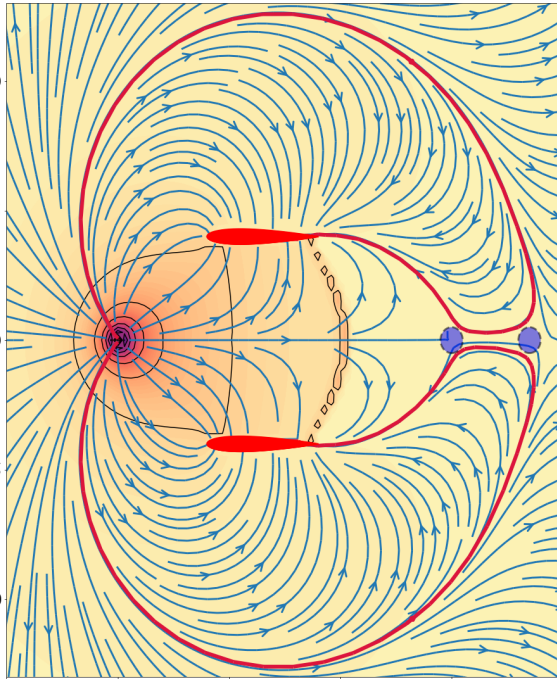
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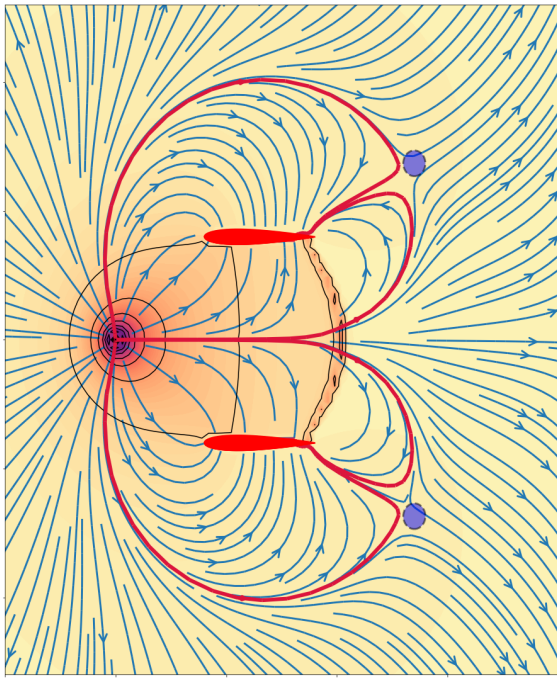
$U_{in} = 2.87 \text{ m/s}$



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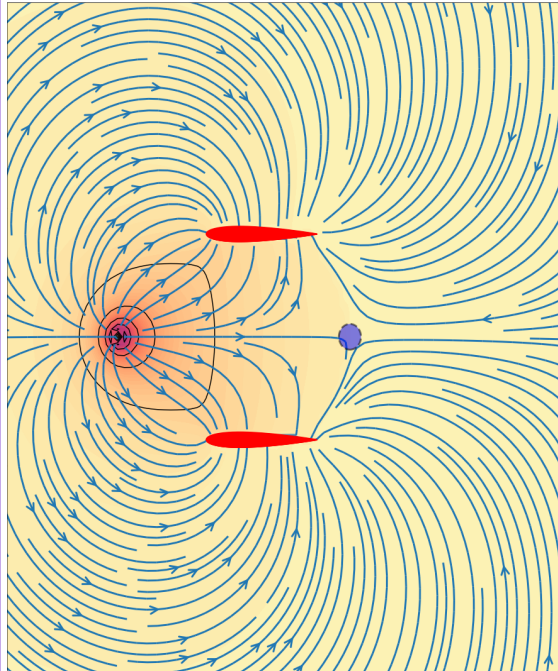


$U_{in} = 12.26 \text{ m/s}$

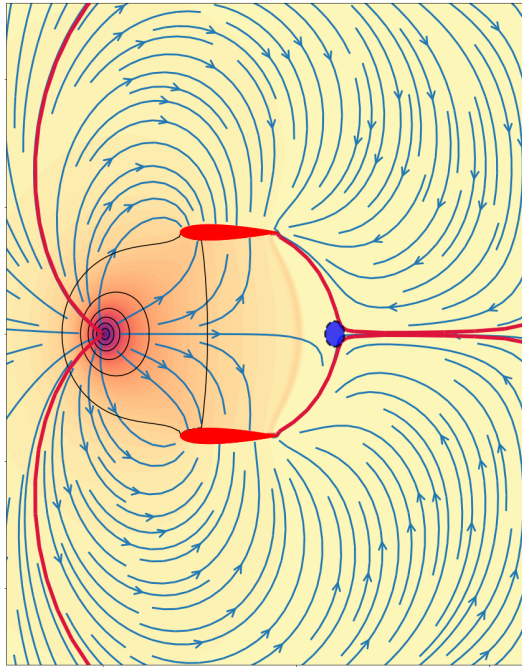


# Electric Field Lines

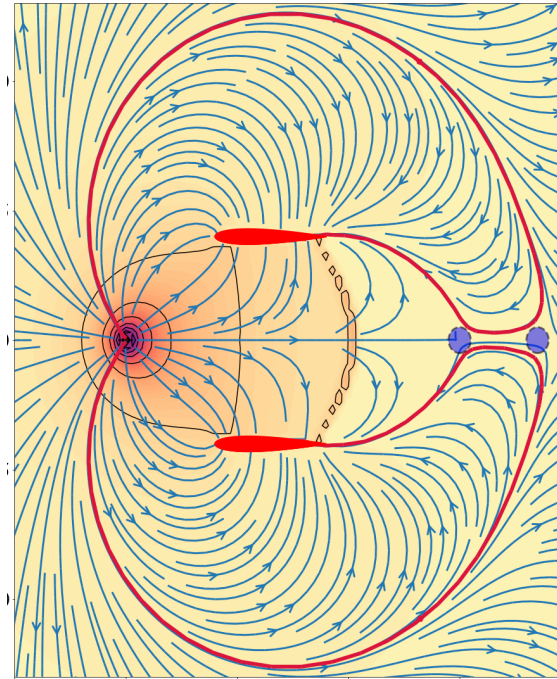
$U_{in} = 1 \text{ m/s}$



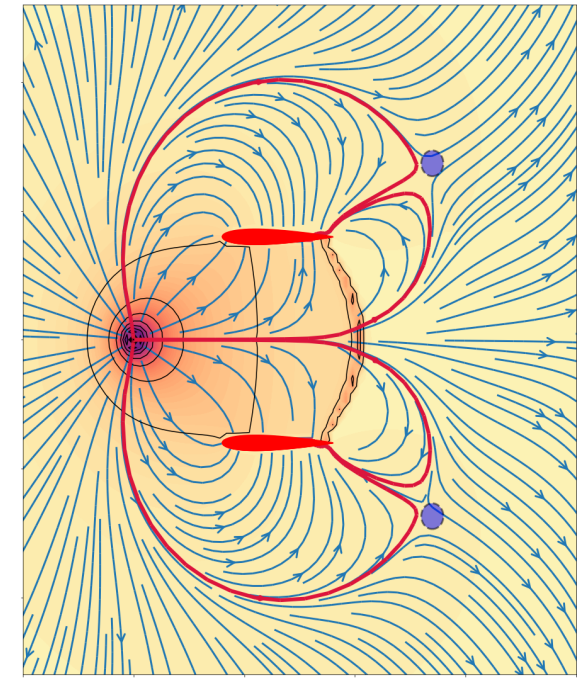
$U_{in} = 2.87 \text{ m/s}$



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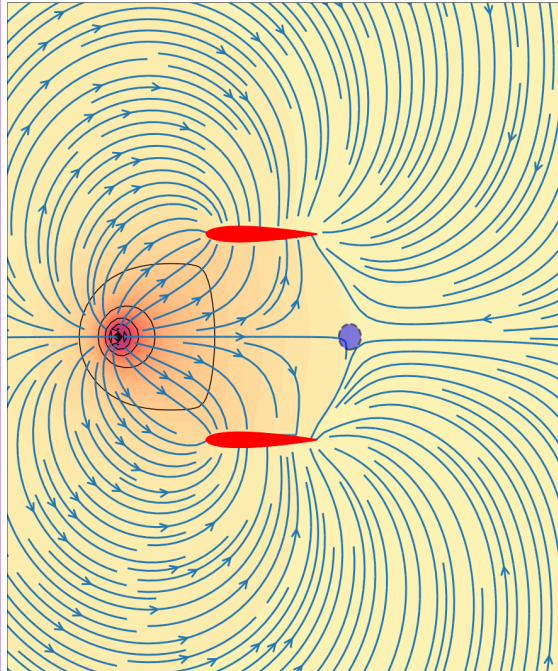
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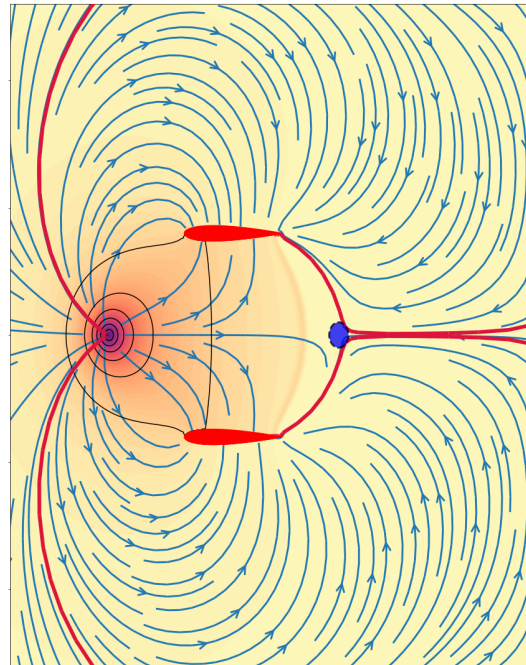
⇒ Decreasing **charge collection zone**

# Electric Field Lines

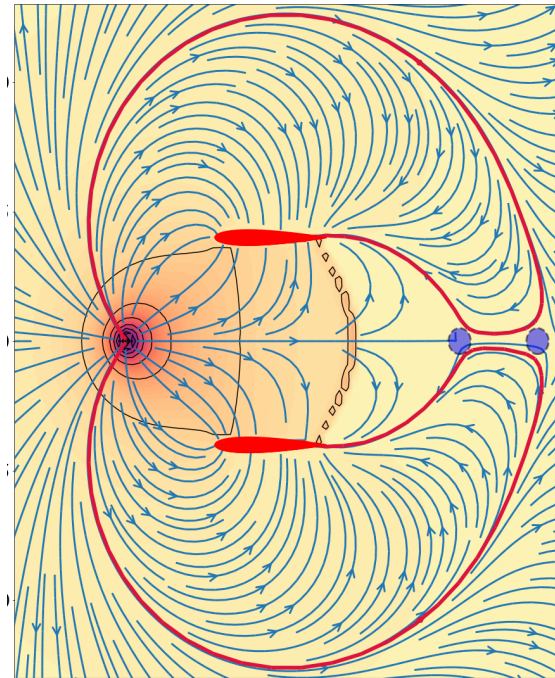
$U_{in} = 1 \text{ m/s}$



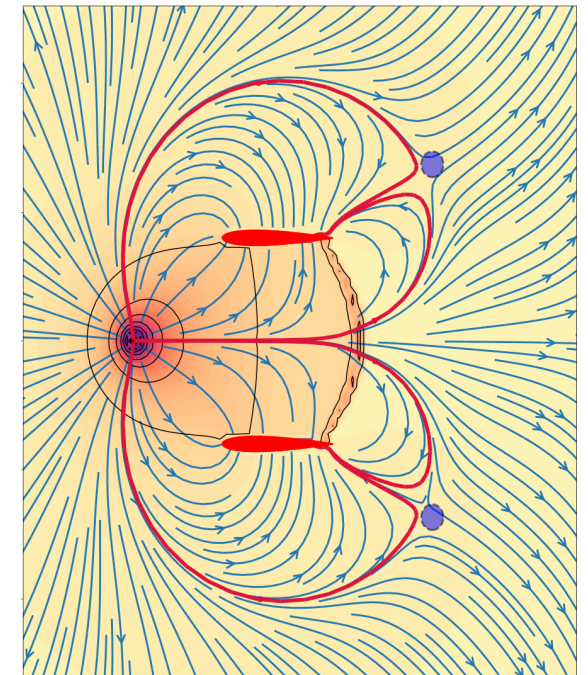
$U_{in} = 2.87 \text{ m/s}$



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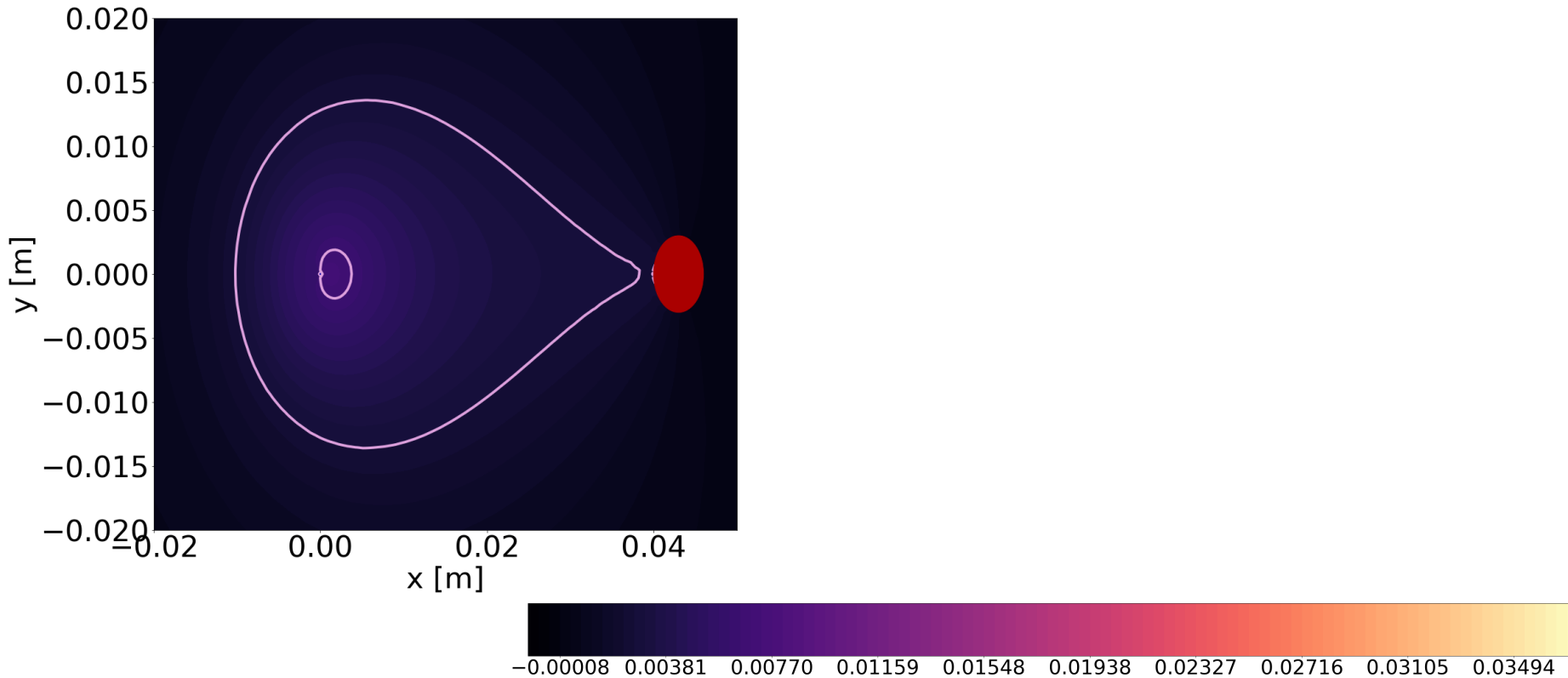


⇒ Decreasing **charge collection zone**

⇒ Ion **boundary layer**

# Ion Density Distributions

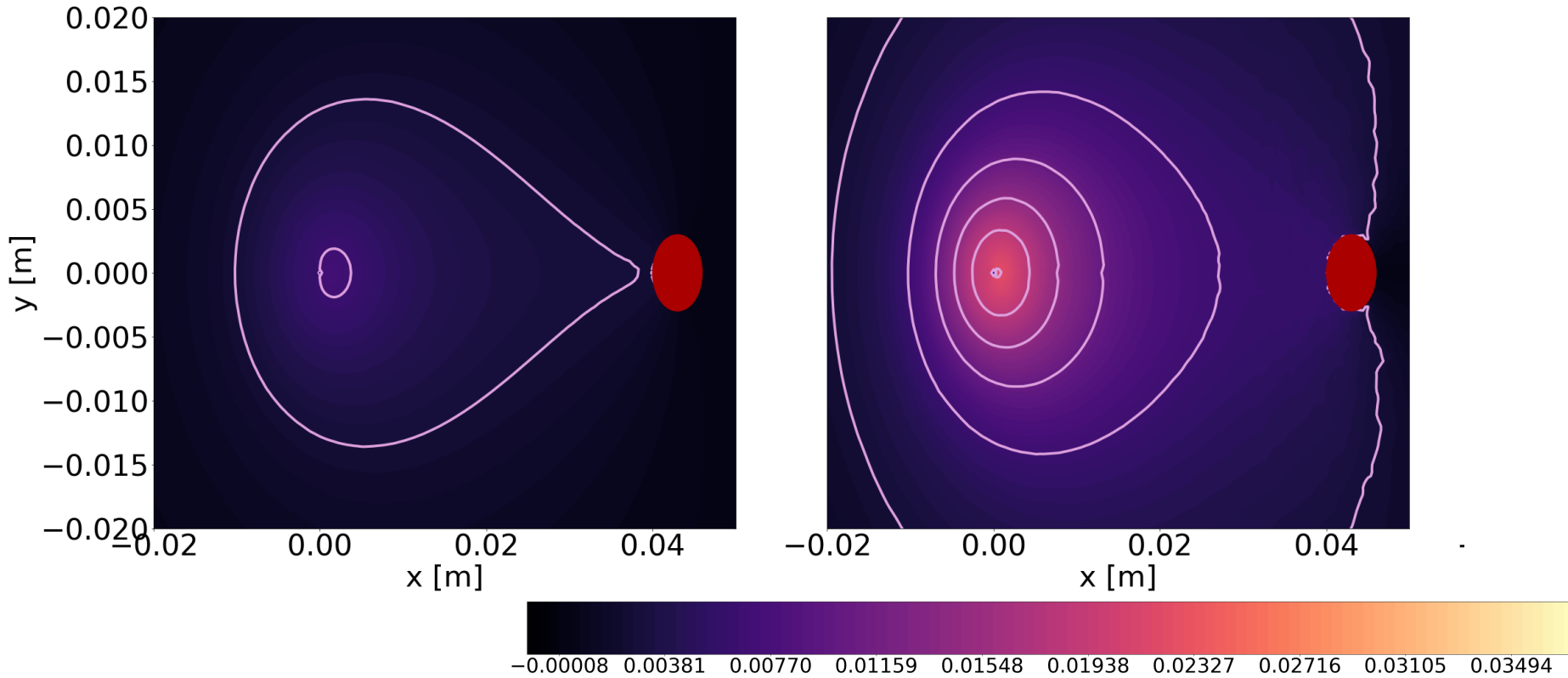
$$U_{in} = 1 \text{ m/s}$$



# Ion Density Distributions

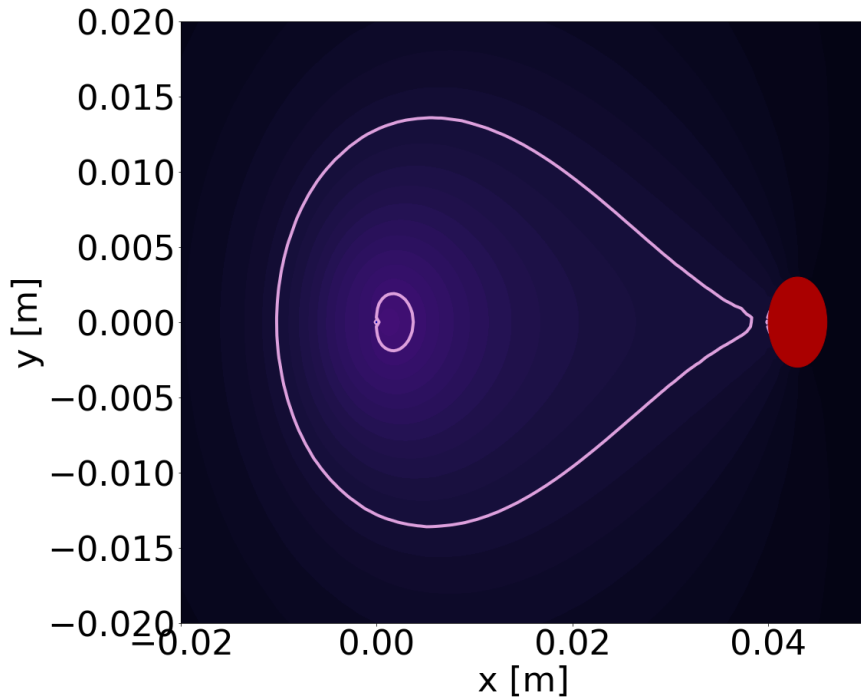
$U_{in} = 1 \text{ m/s}$

$U_{in} = 24 \text{ m/s}$

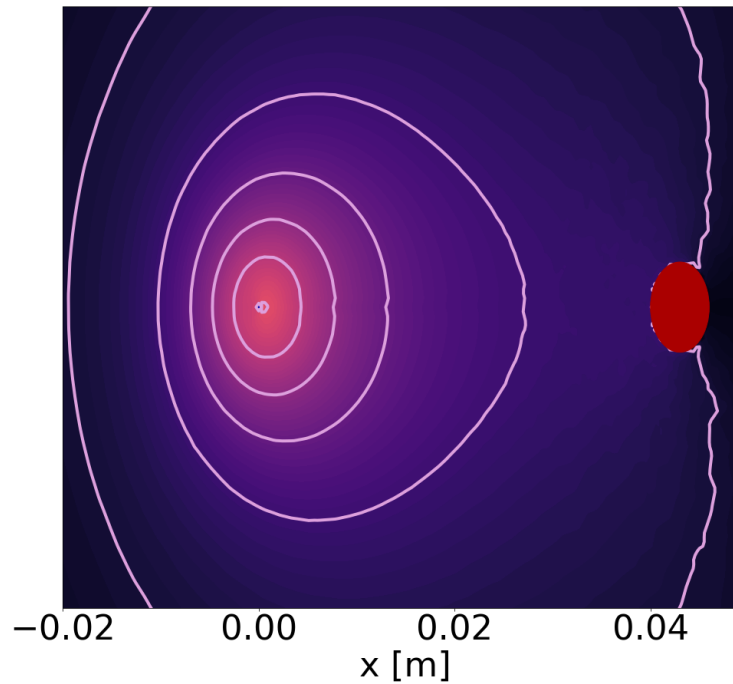


# Ion Density Distributions

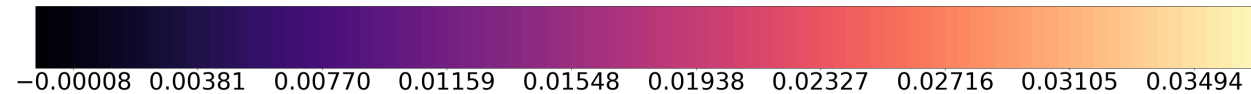
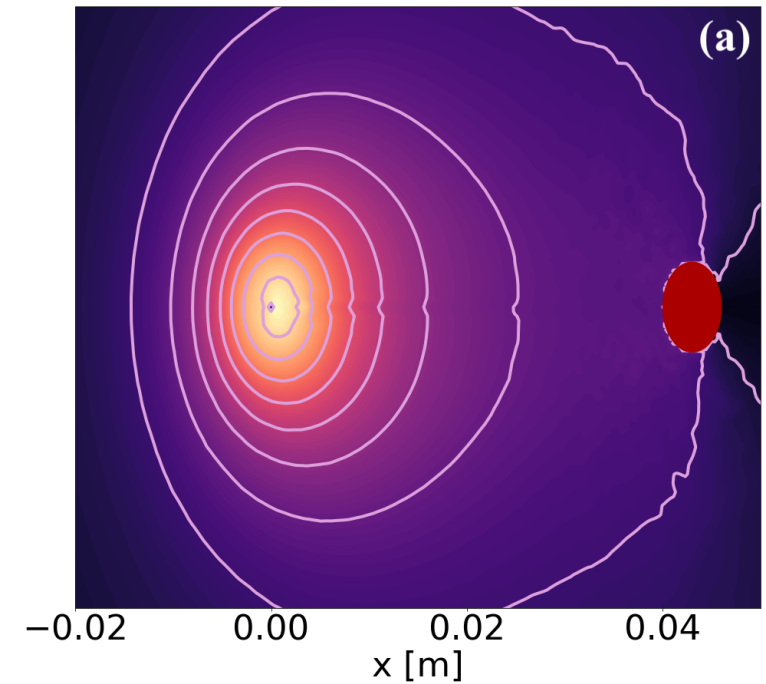
$U_{in} = 1 \text{ m/s}$



$U_{in} = 24 \text{ m/s}$

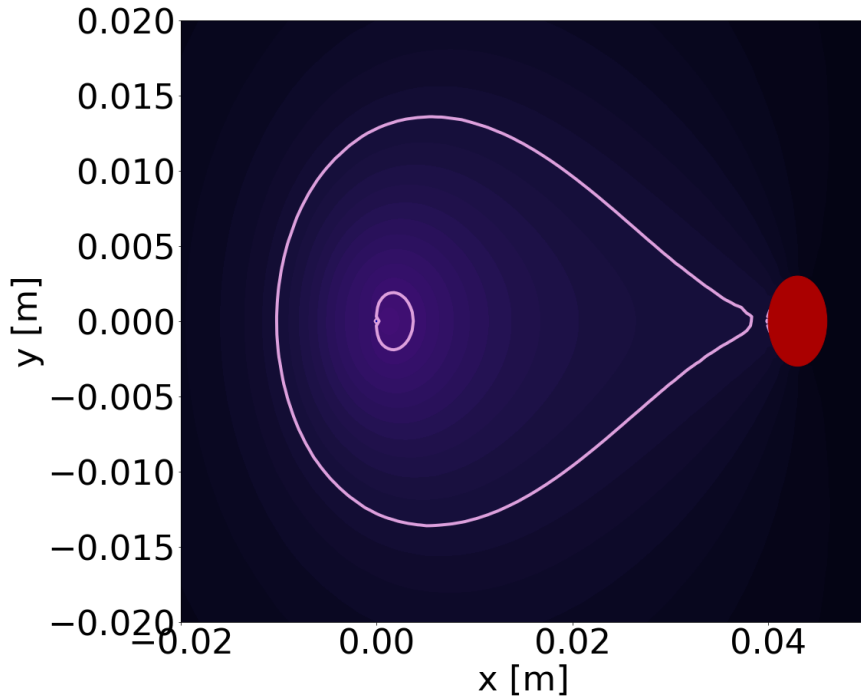


$U_{in} = 49 \text{ m/s}$

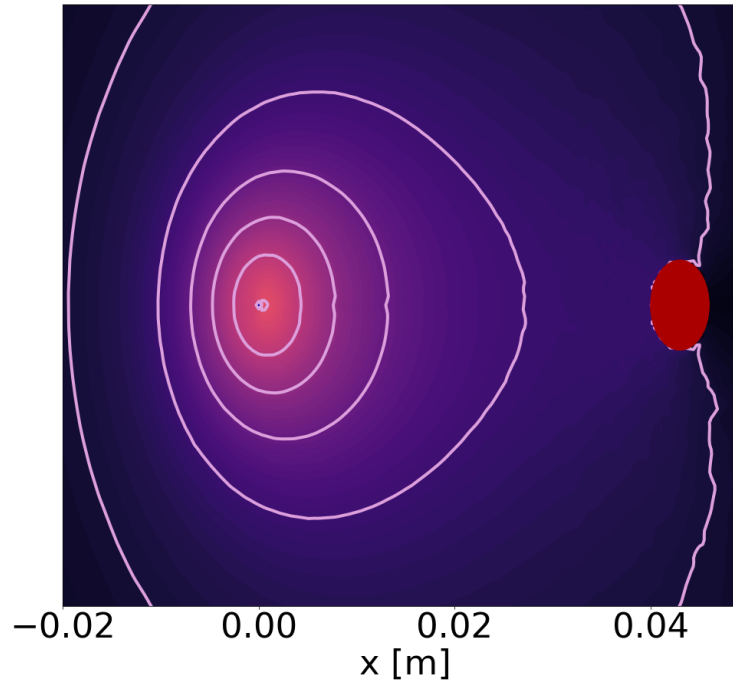


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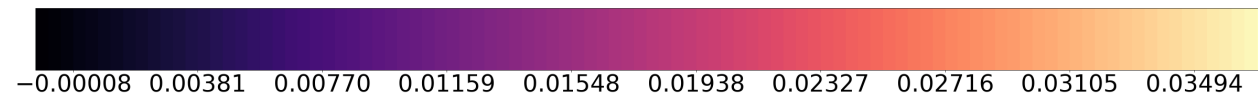
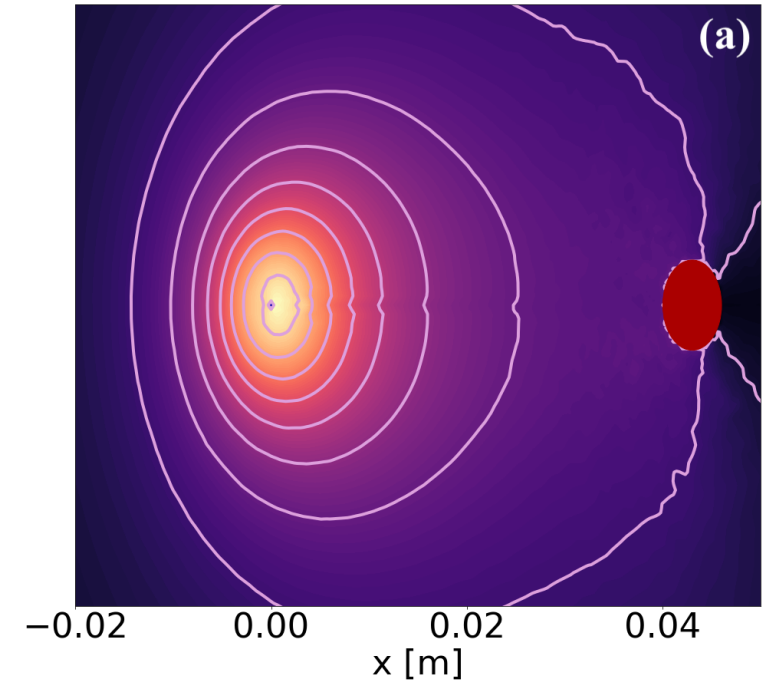
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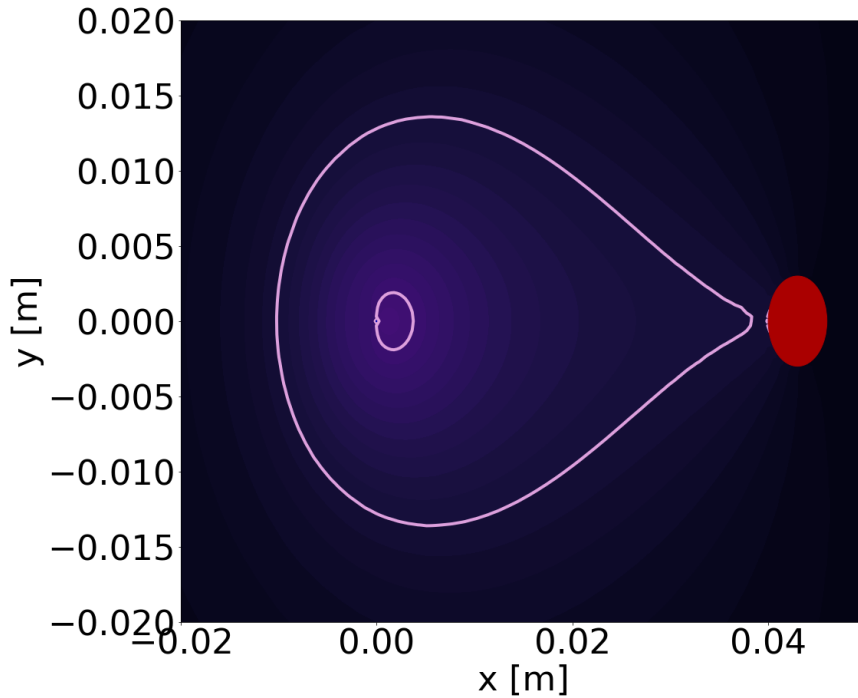
$U_{in} = 49 \text{ m/s}$



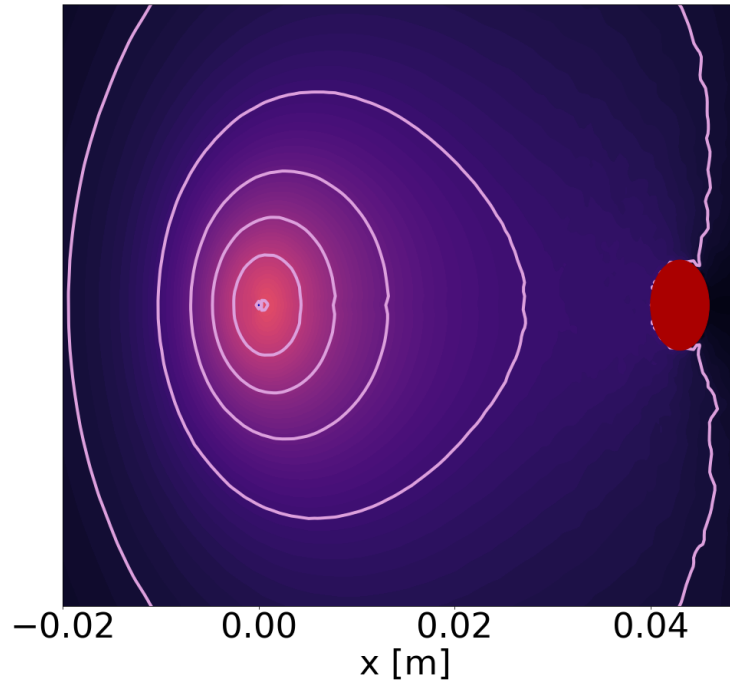
⇒ Ion **density** increase

# Ion Density Distributions

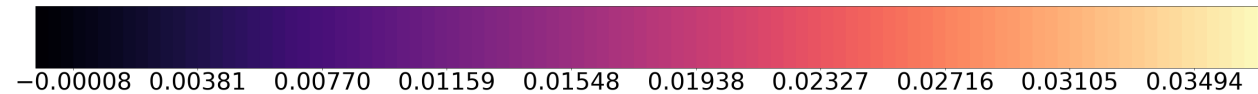
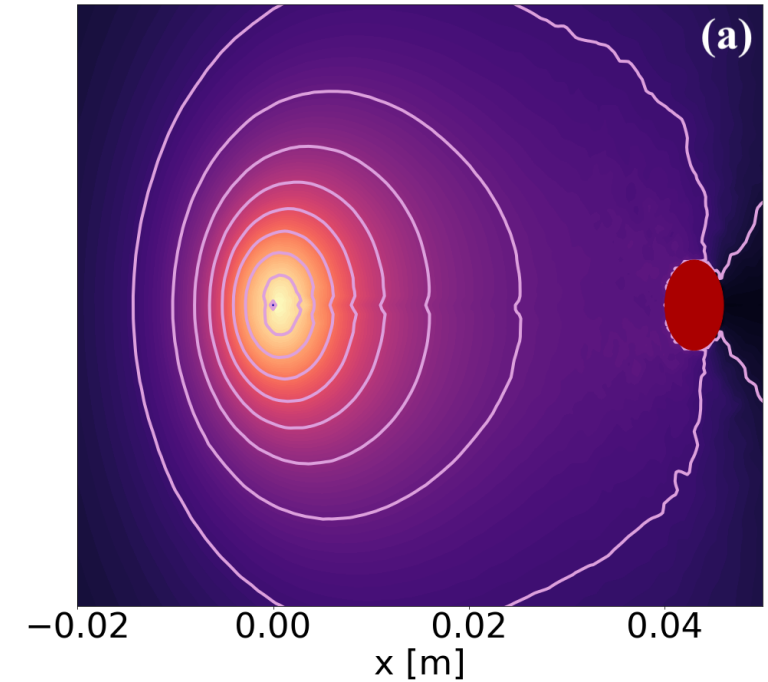
$U_{in} = 1 \text{ m/s}$



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$U_{in} = 49 \text{ m/s}$



⇒ Ion **density** increase, how ?

# Ion Density and Generation Increase

$$\left\{ \begin{array}{l} \Delta\varphi = \delta_{\mu_n} n_n + \delta_{\mu_e} n_e - n_p \\ \nabla \cdot [n_p(-\nabla\varphi + M_m \mathbf{u})] = \alpha \|n_e \mathbf{E}\| \\ \nabla \cdot [n_e(\nabla\varphi + \delta_{\mu} M_m \mathbf{u})] = (\alpha - \eta) \|n_e \mathbf{E}\| \\ \nabla \cdot [n_n(\nabla\varphi + M_m \mathbf{u})] = \eta \|n_e \mathbf{E}\| \end{array} \right.$$



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$$\alpha \propto BN \exp^{-\frac{C}{\|E/N\|}}$$

# Ion Density and Generation Increase

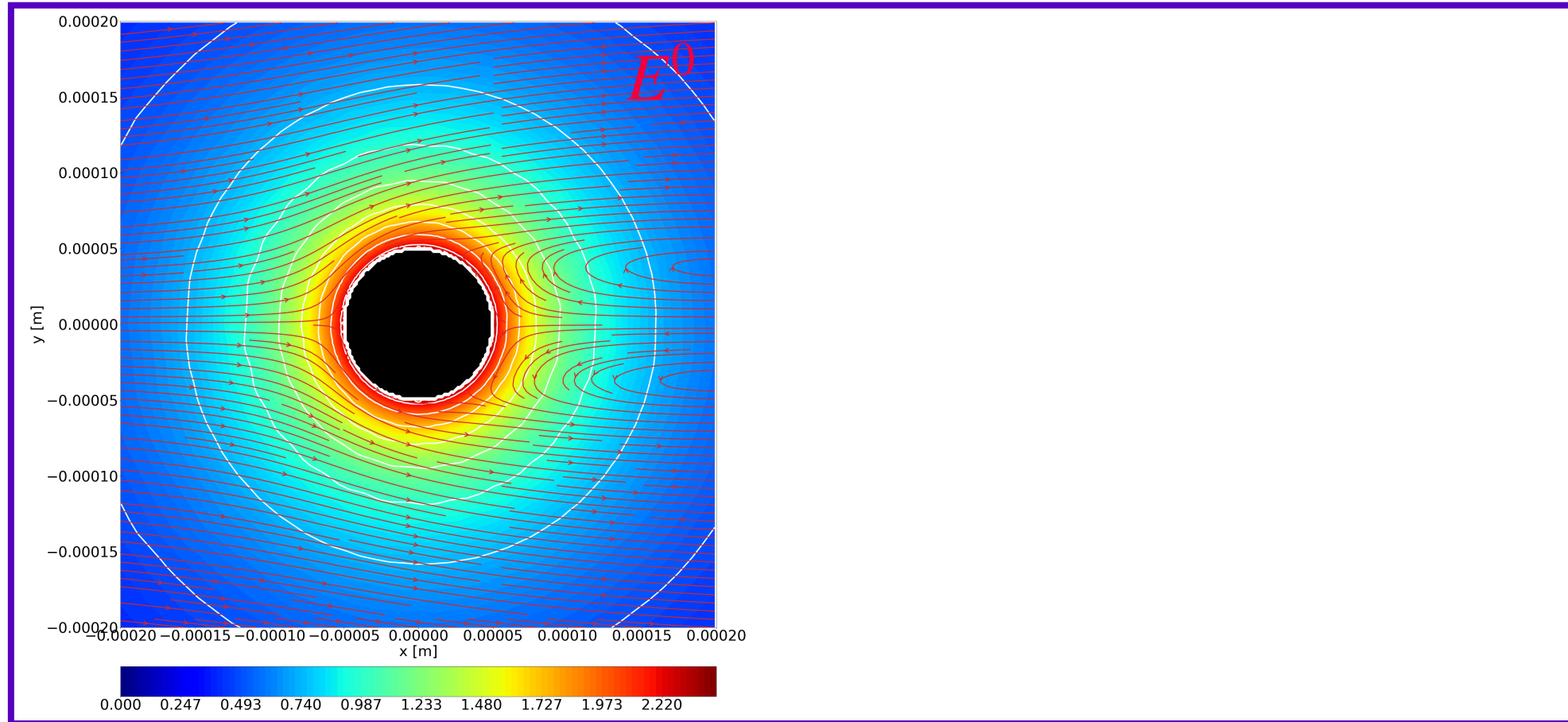
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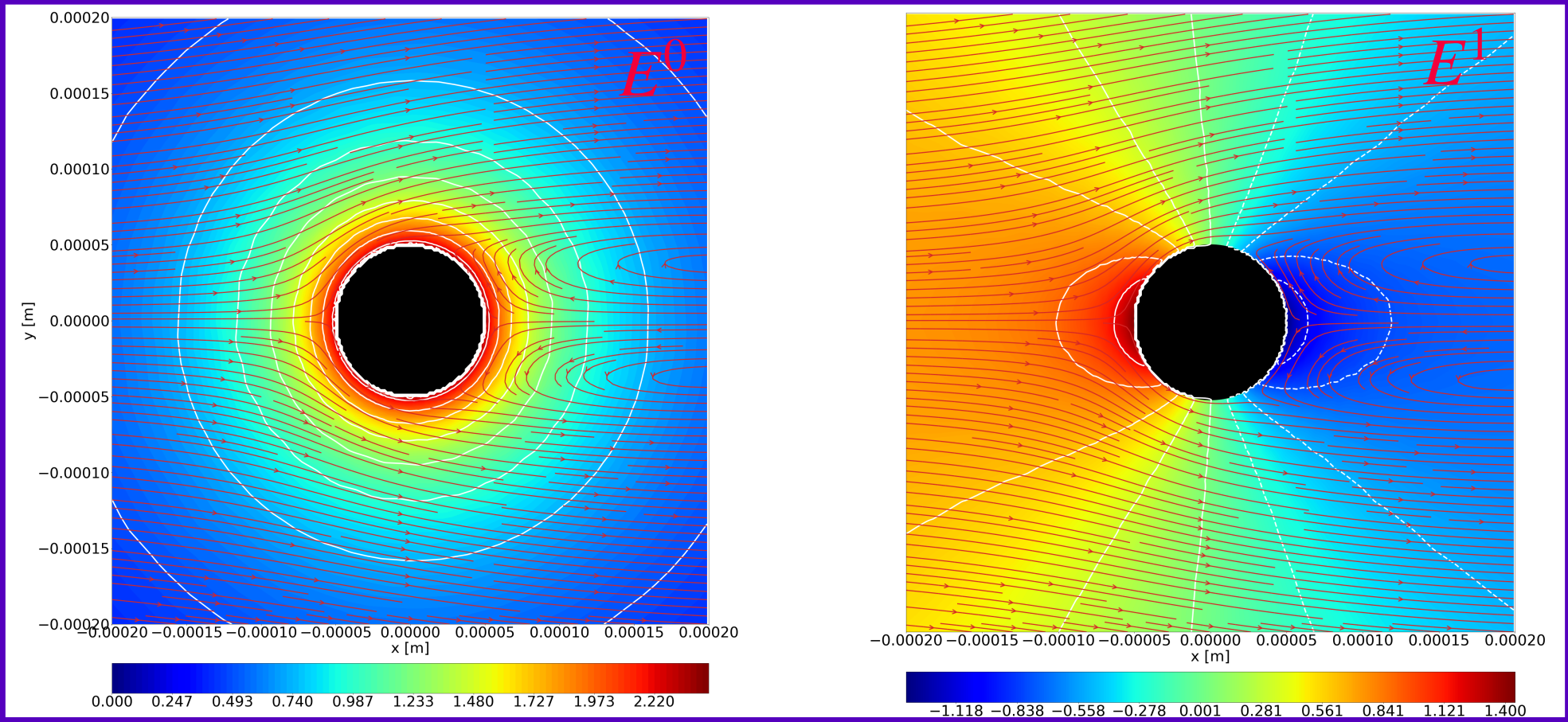
- Charge **generation** increases &  $\alpha = \alpha(\|\mathbf{E}\|) \Rightarrow \|\mathbf{E}\|$  must increase

# Electric Field Enhancement around the Emitter



Base  $\|E\|$

# Electric Field Enhancement around the Emitter

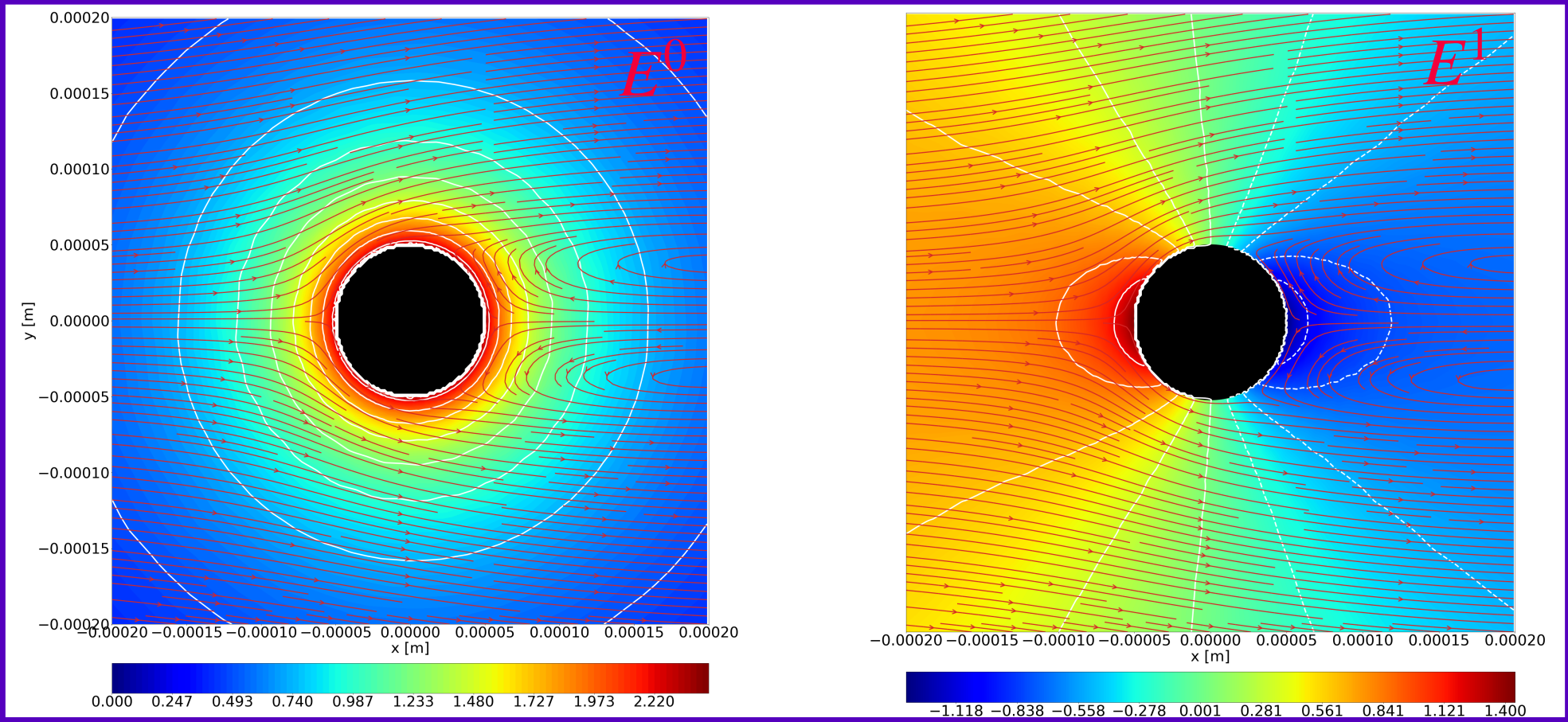


Base  $\|E\|$

$\|E\|$  Perturbation

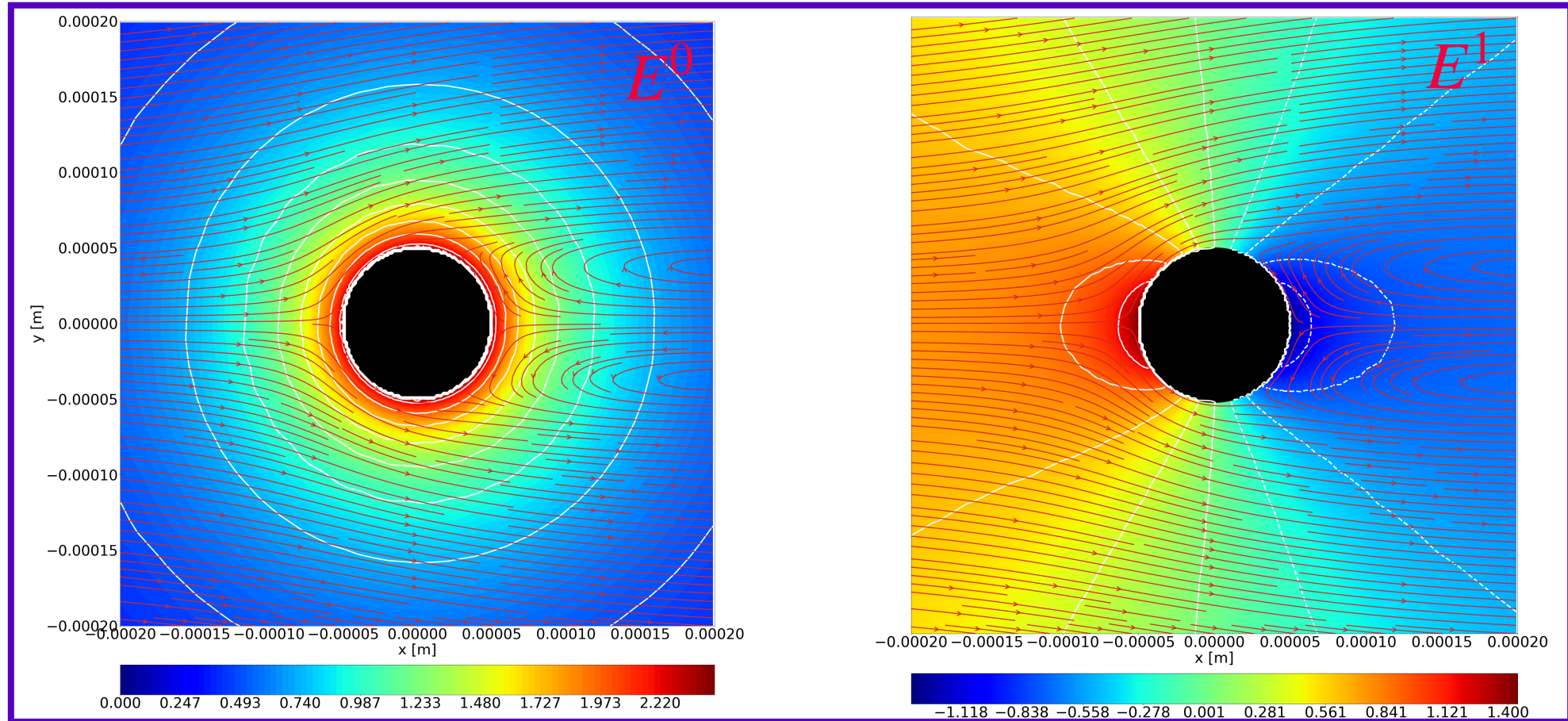


# Electric Field Enhancement around the Emitter



⇒ Advected charges **locally enhance**  $\|E\|$

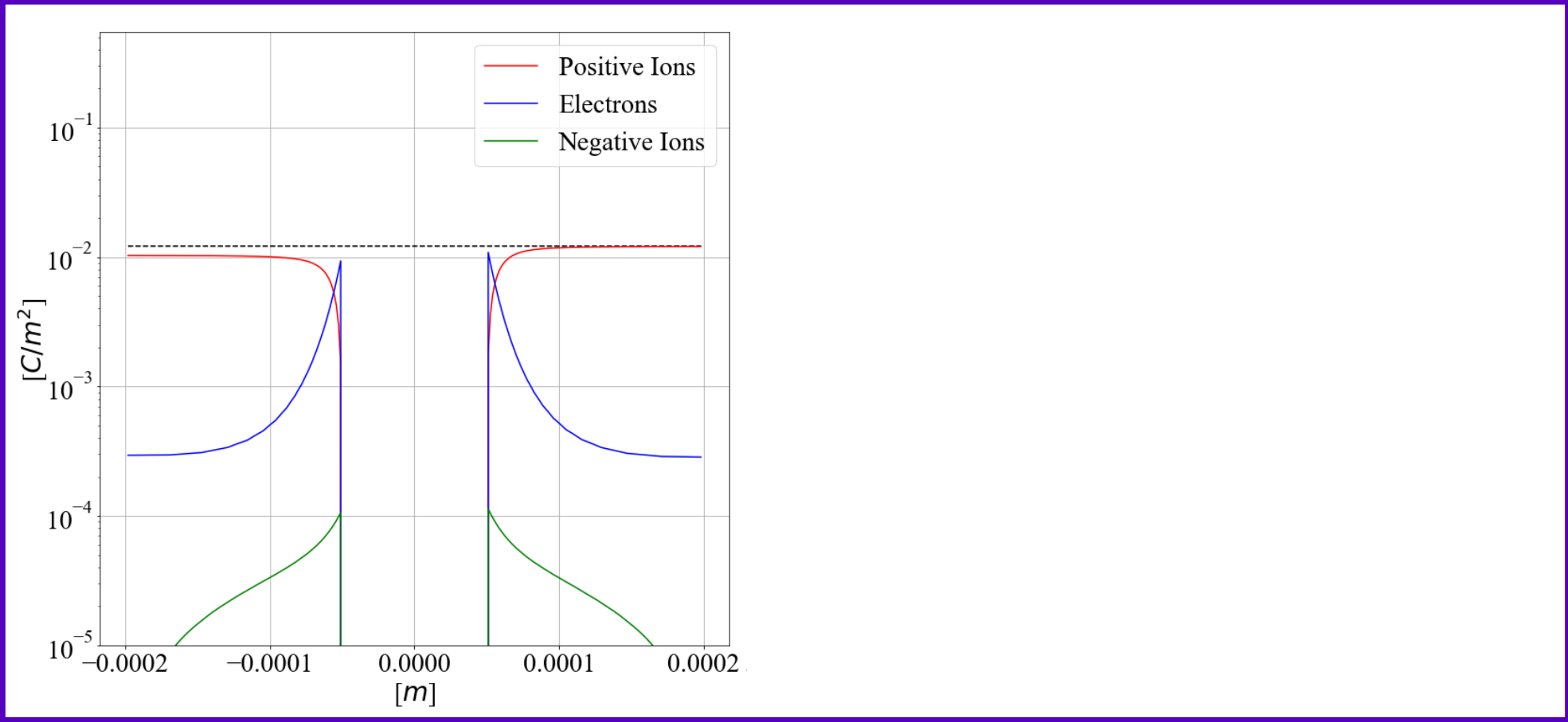
# Electric Field Enhancement around the Emitter



⇒ Advected charges **locally enhance**  $\|\mathbf{E}\|$

⇒ Ion **generation** increases :  $\|\mathbf{E}\| \nearrow \Rightarrow \alpha(\|\mathbf{E}\|) \nearrow$

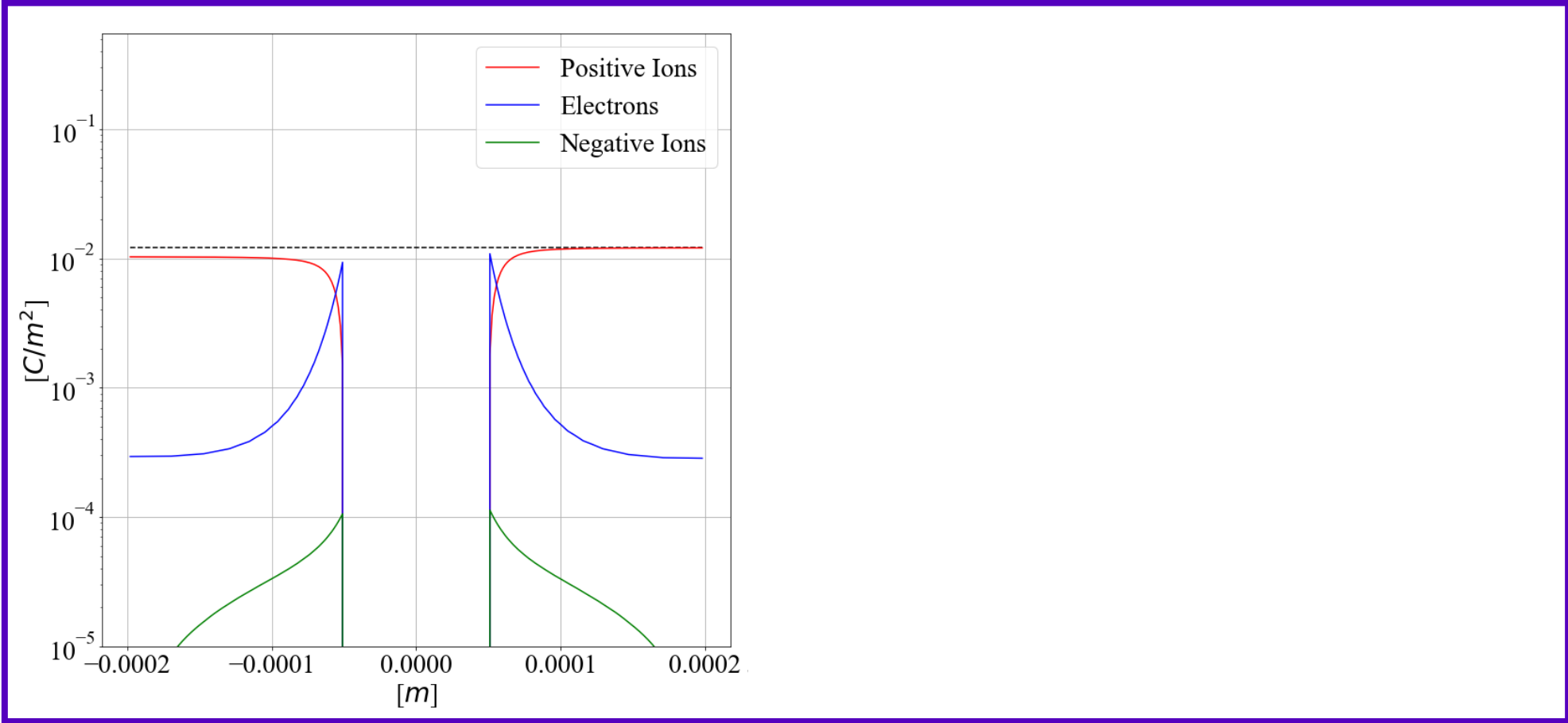
# Charge Profiles around the Emitter



$U_{in} = 0 \text{ m/s}$



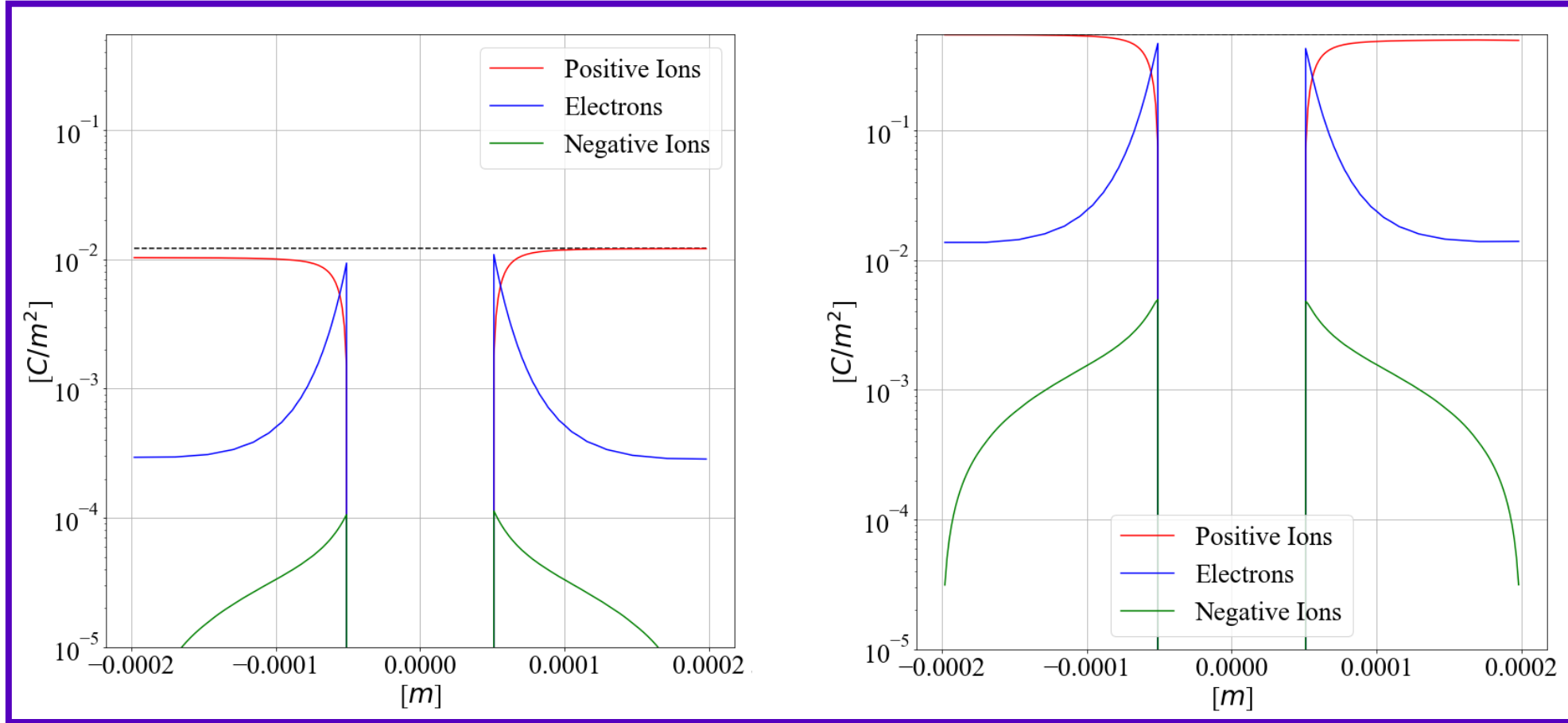
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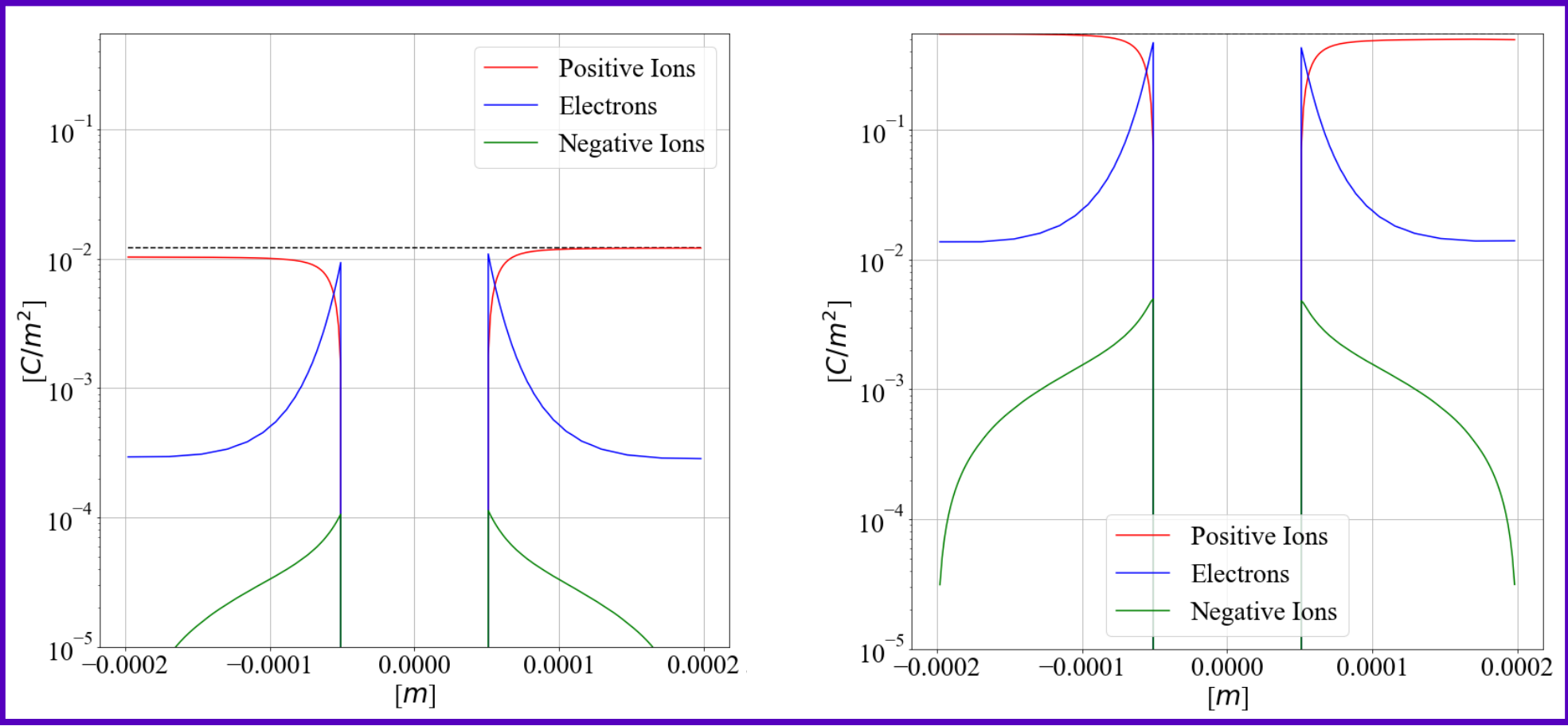


$U_{in} = 0 \text{ m/s}$

$U_{in} = 12 \text{ m/s}$

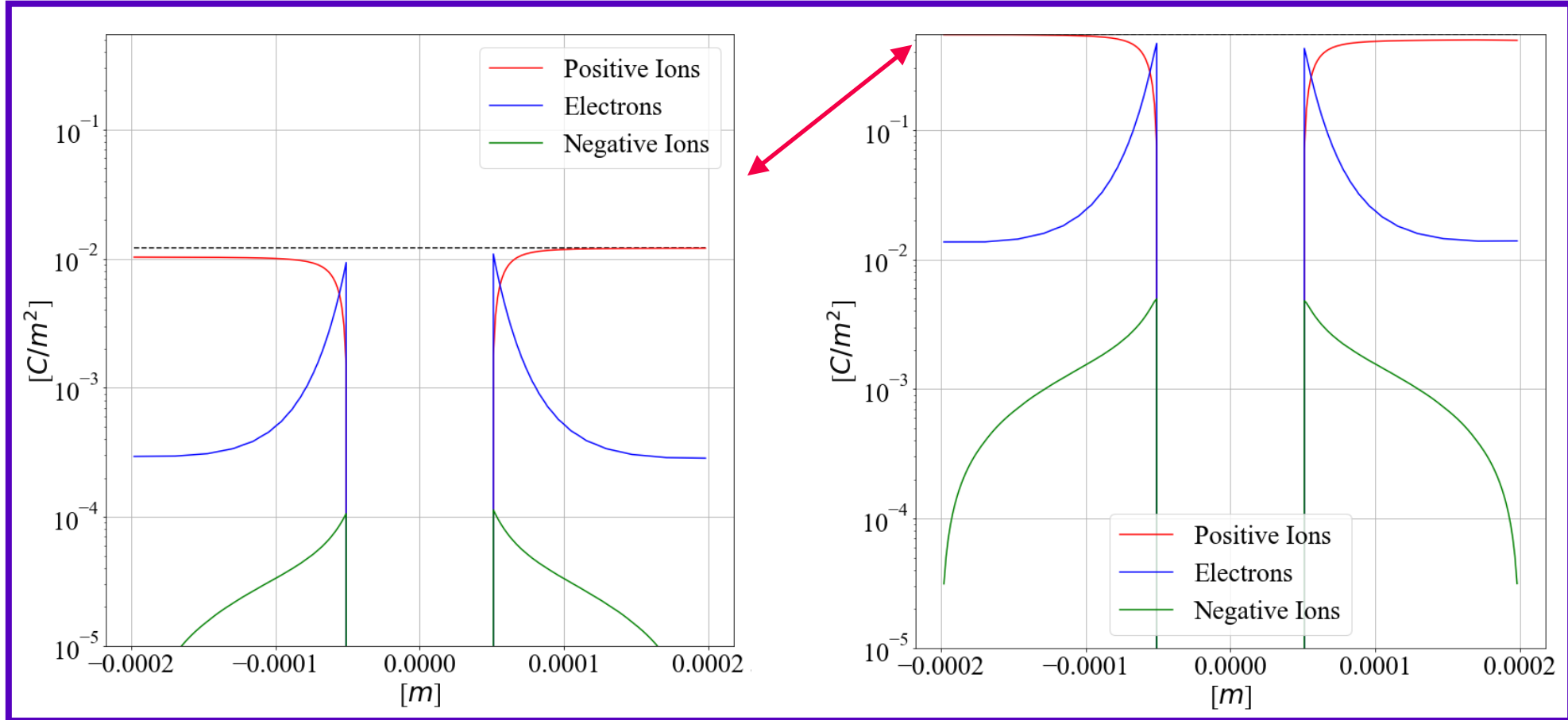
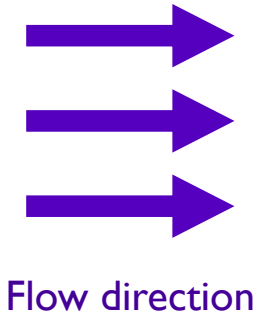


# Charge Profiles around the Emitter



⇒ Charge **densities** increase

# Charge Profiles around the Emitter



Collector

⇒ Charge **densities** increase

⇒ **Symmetry shift** around the emitter



# Conclusion

- **New method:** plasma-fluid interactions accounting with external flow



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- Useful simulations: **complicated experimentally**



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- **Accepted manuscript** : Marques et al 2025 *J. Phys. D:Appl. Phys.*

<https://doi.org/10.1088/1361-6463/ae07f9>



# Thank you !

GECAPS 2025, Seoul, South Korea

J. M. D. C. Marques, D. Fabre, F. Plouraboué

IPROP EIC Pathfinder

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Council



**Funded by  
the European Union**

# Two-Way Model (More) Numerical Aspects

$$L_{NS}(\mathbf{u}; \mathbf{u}, p) := -\frac{1}{Re} \Delta \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p + n_p \nabla \varphi$$



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$$\Rightarrow \int_{\Omega} L_{NS}(\mathbf{u}; \mathbf{u}, p) L_{NS}(\mathbf{v}, q)^\dagger d\Omega + \text{BC}$$



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SUPG/PSPG  
(Artificial diffusion)

$$+ \sum_{t \in T_h} \int_{T_h} L_{NS}(\mathbf{u}_h; \mathbf{u}_h, p_h) \cdot \tau_{SUPG}((\mathbf{u}_h \cdot \nabla) \mathbf{v}_h + \nabla q_h)$$



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$$+ \sum_{t \in T_h} \int_{T_h} L_{NS}(\mathbf{u}_h; \mathbf{u}_h, p_h) \cdot \underbrace{\tau_{SUPG}((\mathbf{u}_h \cdot \nabla) \mathbf{v}_h + \nabla q_h)}_{\text{Small local viscosity correction on the upstream direction}}$$



# Two-Way Model (More) Numerical Aspects

$$L_{NS}(\mathbf{u}; \mathbf{u}, p) := -\frac{1}{Re} \Delta \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p + n_p \nabla \varphi$$

$$\Rightarrow \int_{\Omega} L_{NS}(\mathbf{u}; \mathbf{u}, p) L_{NS}(\mathbf{v}, q)^\dagger d\Omega + \text{BC}$$

SUPG/PSPG  
(Artificial diffusion)

$$+ \sum_{t \in T_h} \int_{T_h} L_{NS}(\mathbf{u}_h; \mathbf{u}_h, p_h) \cdot \underbrace{\tau_{SUPG}((\mathbf{u}_h \cdot \nabla) \mathbf{v}_h + \nabla q_h)}_{\text{Small local viscosity correction on the upstream direction}}$$

Grad-div  
(Numerical mass conservation)

$$+ \sum_{t \in T_h} \int_{T_h} (\nabla \cdot \mathbf{u}_h) \gamma_{grad-div} (\nabla \cdot \mathbf{v}_h)$$



# Two-Way Model (More) Numerical Aspects

$$L_{NS}(\mathbf{u}; \mathbf{u}, p) := -\frac{1}{Re} \Delta \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p + n_p \nabla \varphi$$

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(Artificial diffusion)

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Grad-div  
(Numerical mass conservation)

$$+ \sum_{t \in T_h} \int_{T_h} (\nabla \cdot \mathbf{u}_h) \underbrace{\gamma_{grad-div}(\nabla \cdot \mathbf{v}_h)}_{\text{Imposing locally numerical mass conservation on the test function}}$$



# Two-Way Model (More) Numerical Aspects

$$L_{NS}(\mathbf{u}; \mathbf{u}, p) := -\frac{1}{R_e} \Delta \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p + n_p \nabla \varphi$$

$$\Rightarrow \int_{\Omega} L_{NS}(\mathbf{u}; \mathbf{u}, p) L_{NS}(\mathbf{v}, q)^\dagger d\Omega + \text{BC}$$

SUPG/PSPG  
(Artificial diffusion)

$$+ \sum_{t \in T_h} \int_{T_h} L_{NS}(\mathbf{u}_h; \mathbf{u}_h, p_h) \cdot \underbrace{\tau_{SUPG}((\mathbf{u}_h \cdot \nabla) \mathbf{v}_h + \nabla q_h)}_{\text{Small local viscosity correction on the upstream direction}}$$

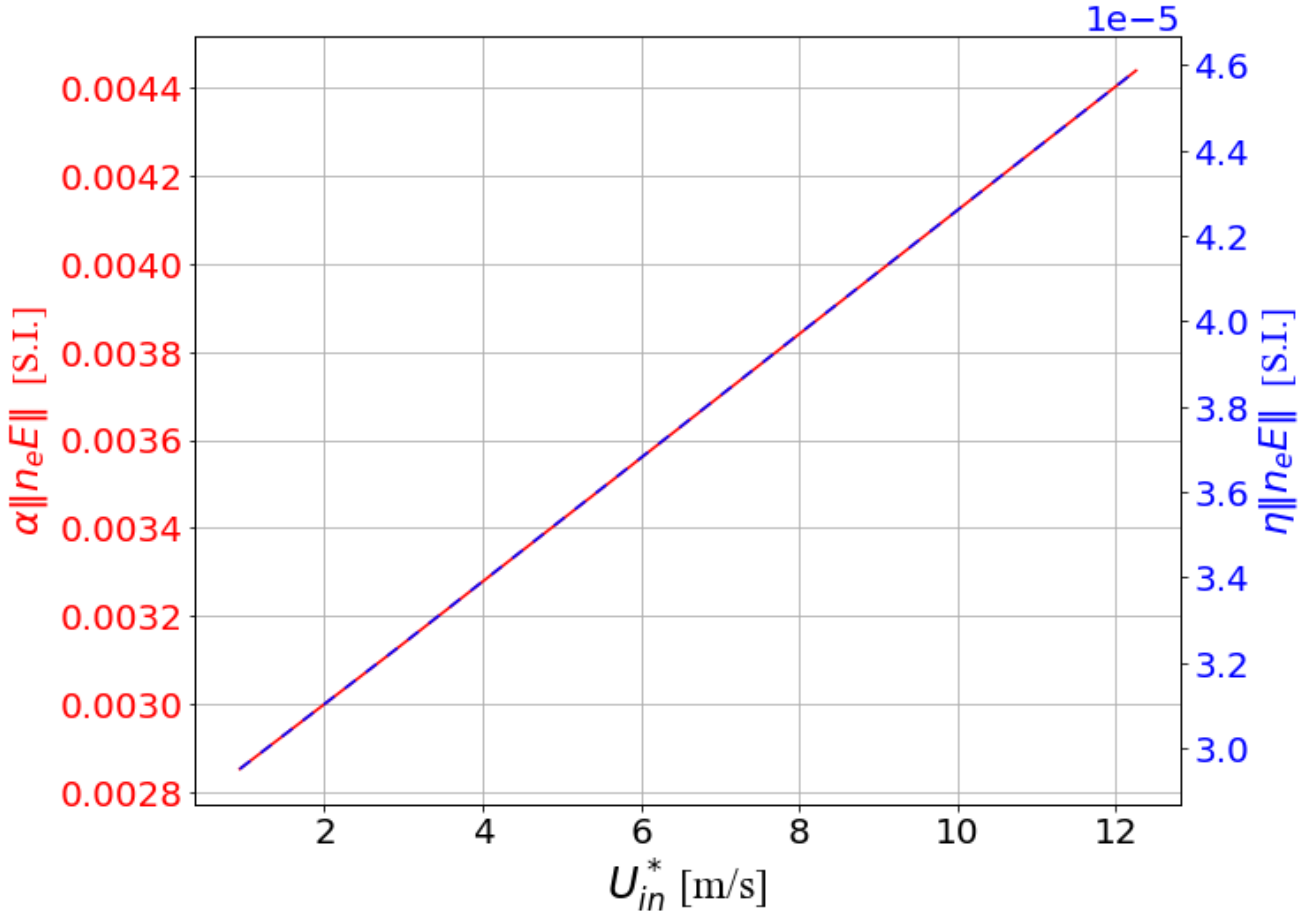
Grad-div  
(Numerical mass  
conservation)

$$+ \sum_{t \in T_h} \int_{T_h} (\nabla \cdot \mathbf{u}_h) \underbrace{\gamma_{grad-div}(\nabla \cdot \mathbf{v}_h)}_{\text{Imposing locally numerical mass conservation on the test function}}$$

$$\tau_{SUPG} = \frac{\delta_{SUPG}}{\sqrt{4 \frac{\|\mathbf{u}^{n-1}_h\|^2}{h_T^2} + \frac{16}{R_{e,fluid}^2 h_T^4}}}$$

$$\gamma_{grad-div} \sim o(1)$$

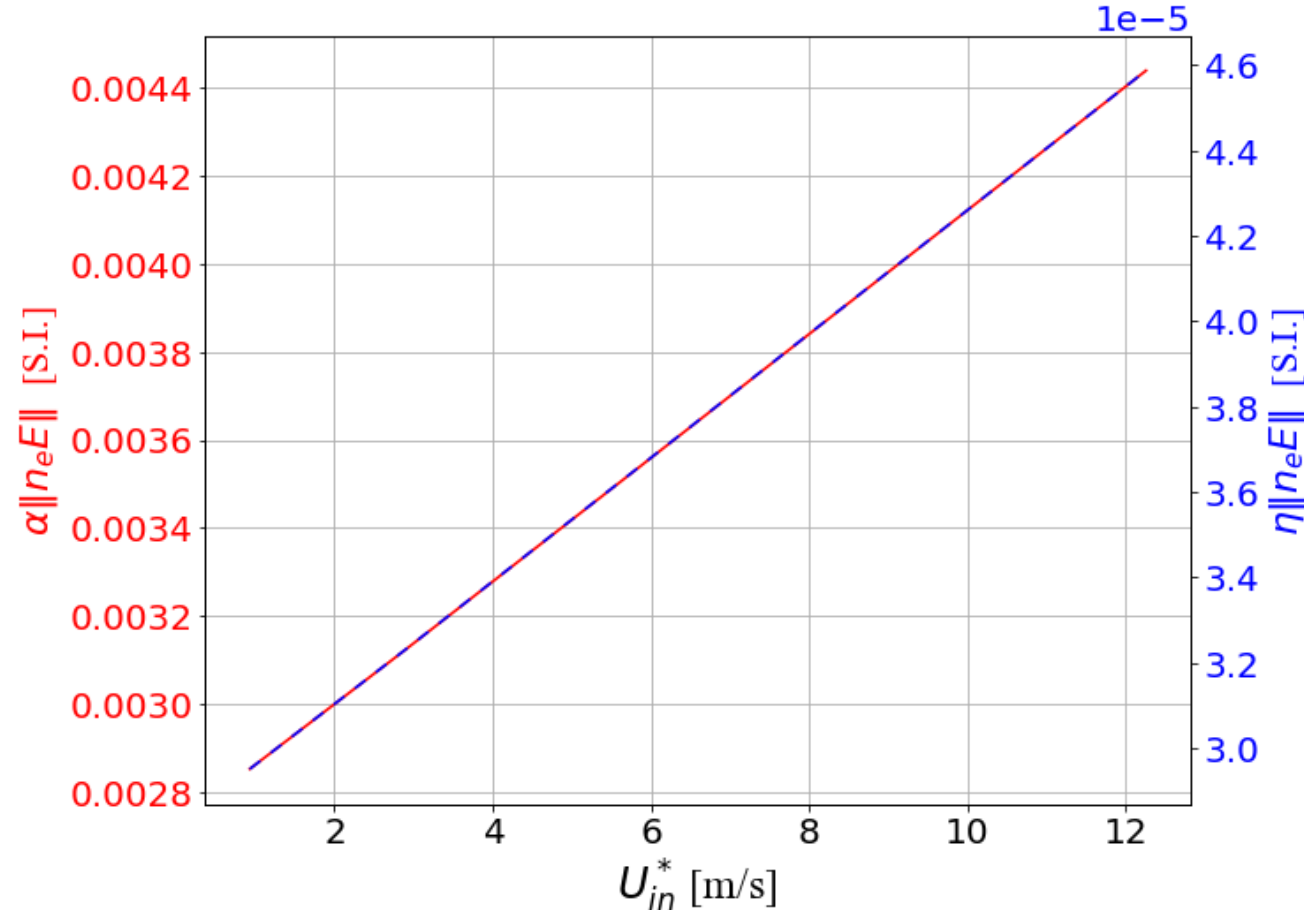
# Townsend Avalanche



$$\alpha \propto BN \exp^{-\frac{C}{\|E/N\|}}$$



# Townsend Avalanche

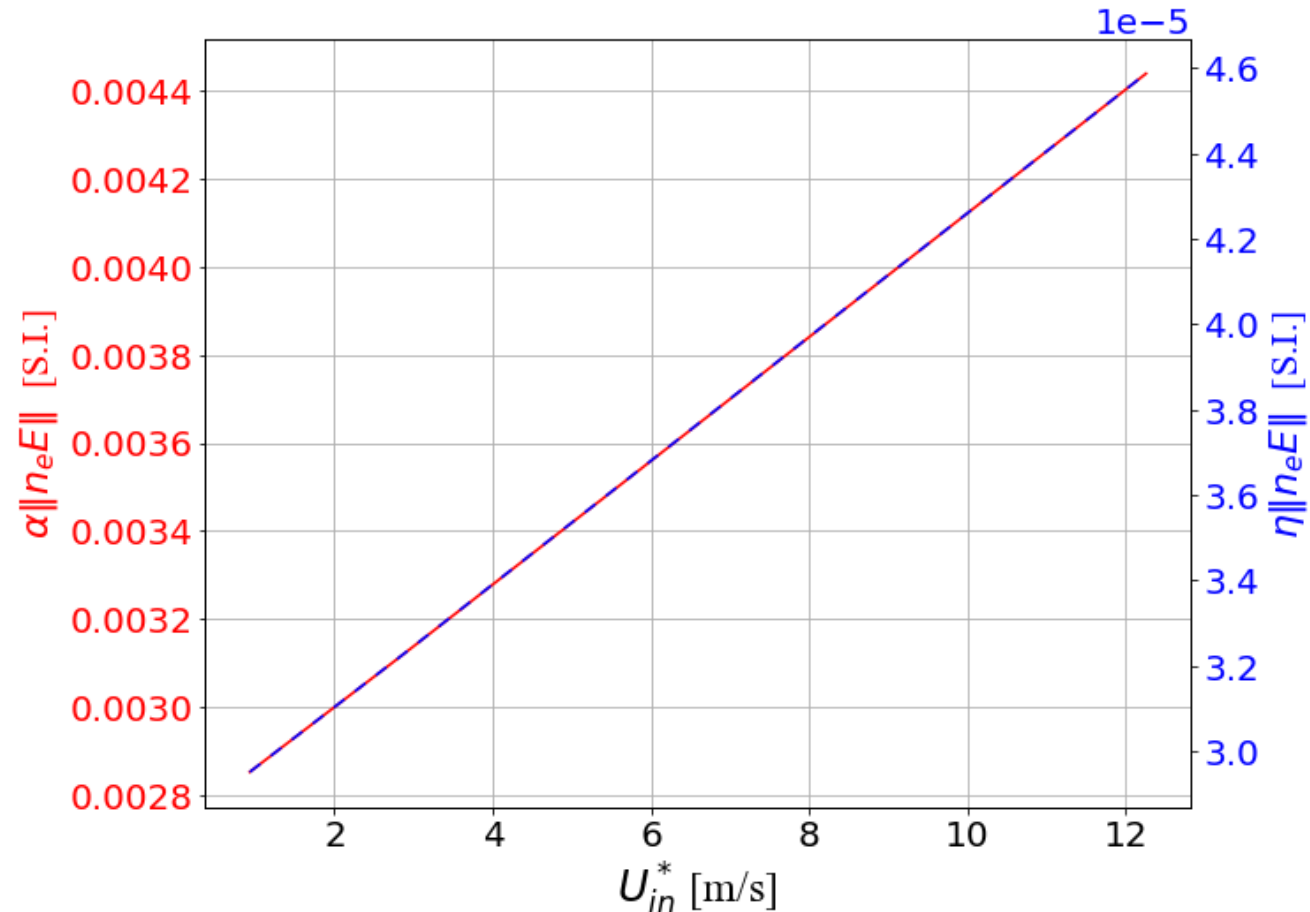


$$\alpha \propto BN \exp^{-\frac{C}{||E/N||}}$$

⇒ Charge **generation** increases



# Townsend Avalanche

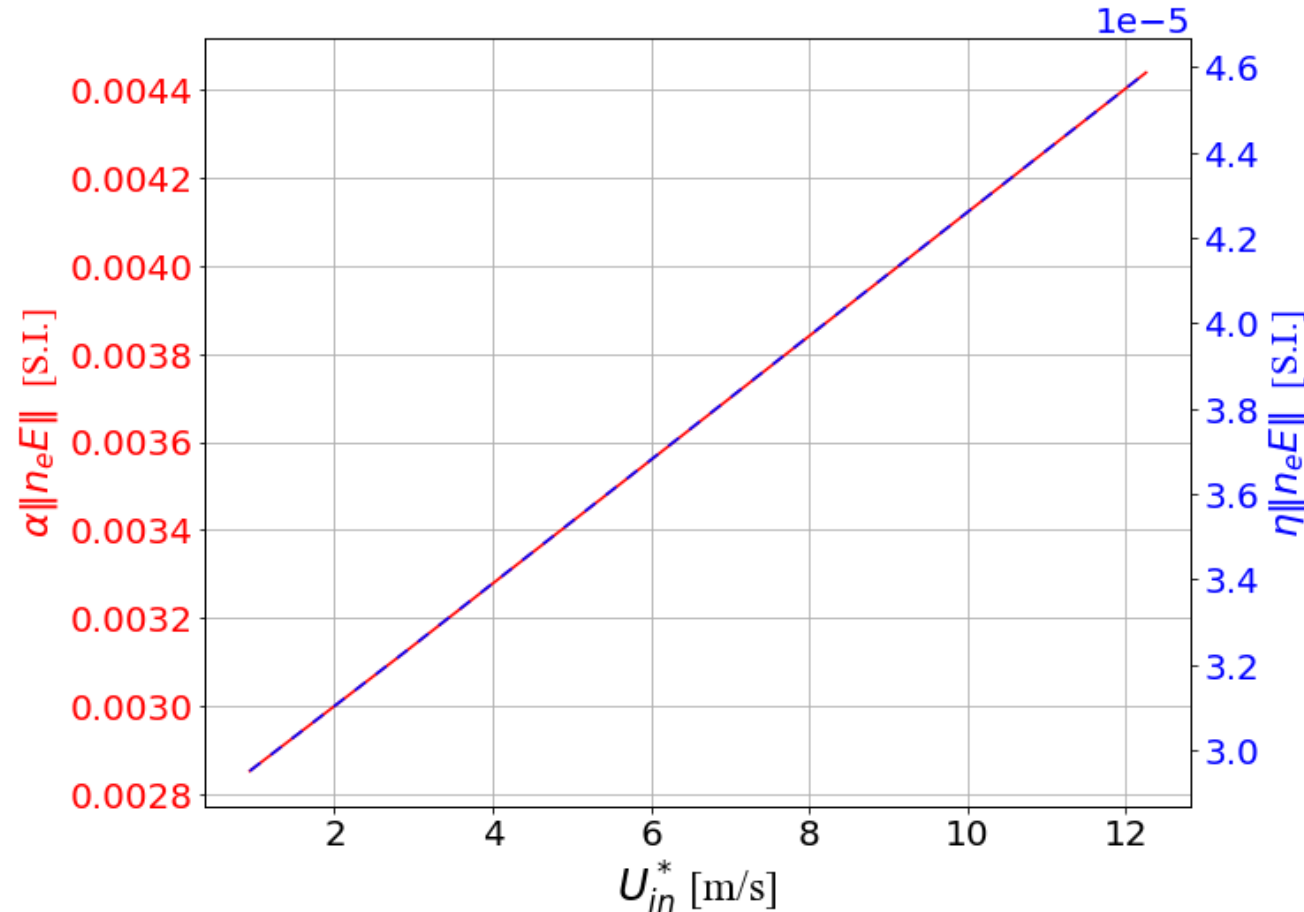


$$\alpha \propto BN \exp^{-\frac{C}{\|E/N\|}}$$

⇒ Charge **generation** increases

⇒ Depends only on  $\|E\|$  :  $\alpha = \alpha(\|E\|)$

# Townsend Avalanche

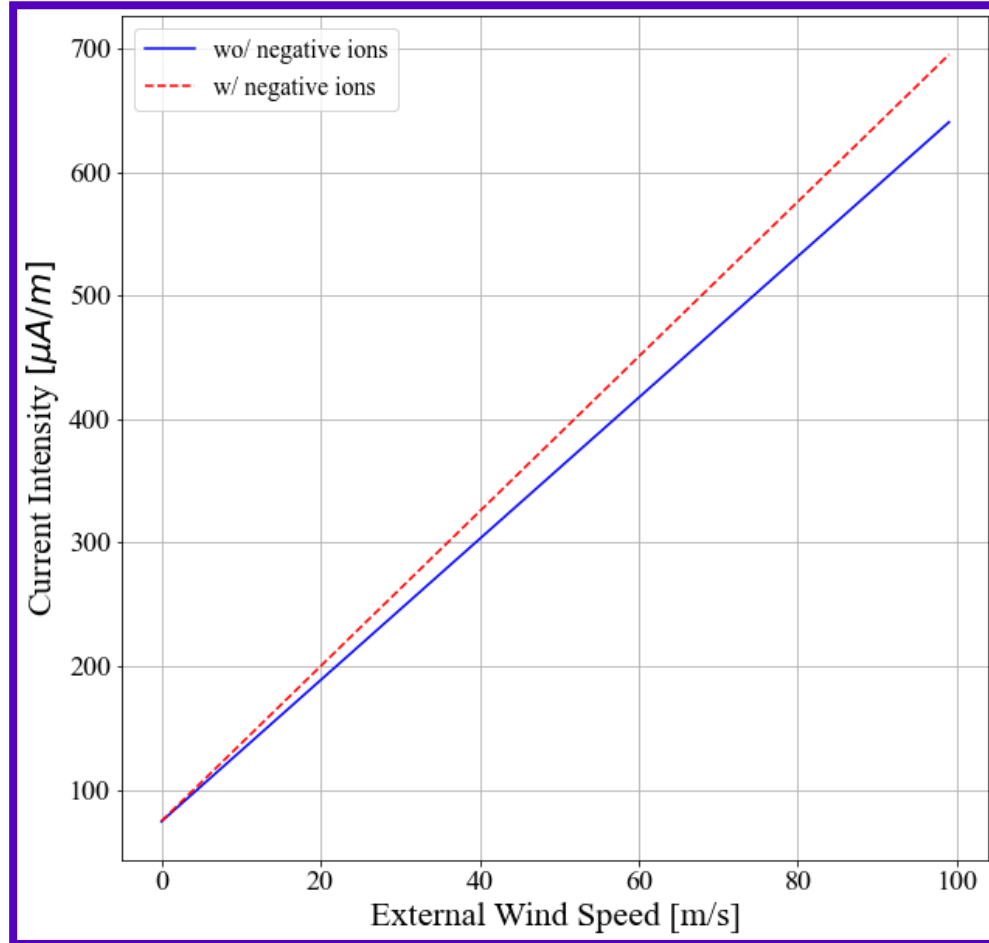


$$\alpha \propto BN \exp^{-\frac{C}{||E/N||}}$$

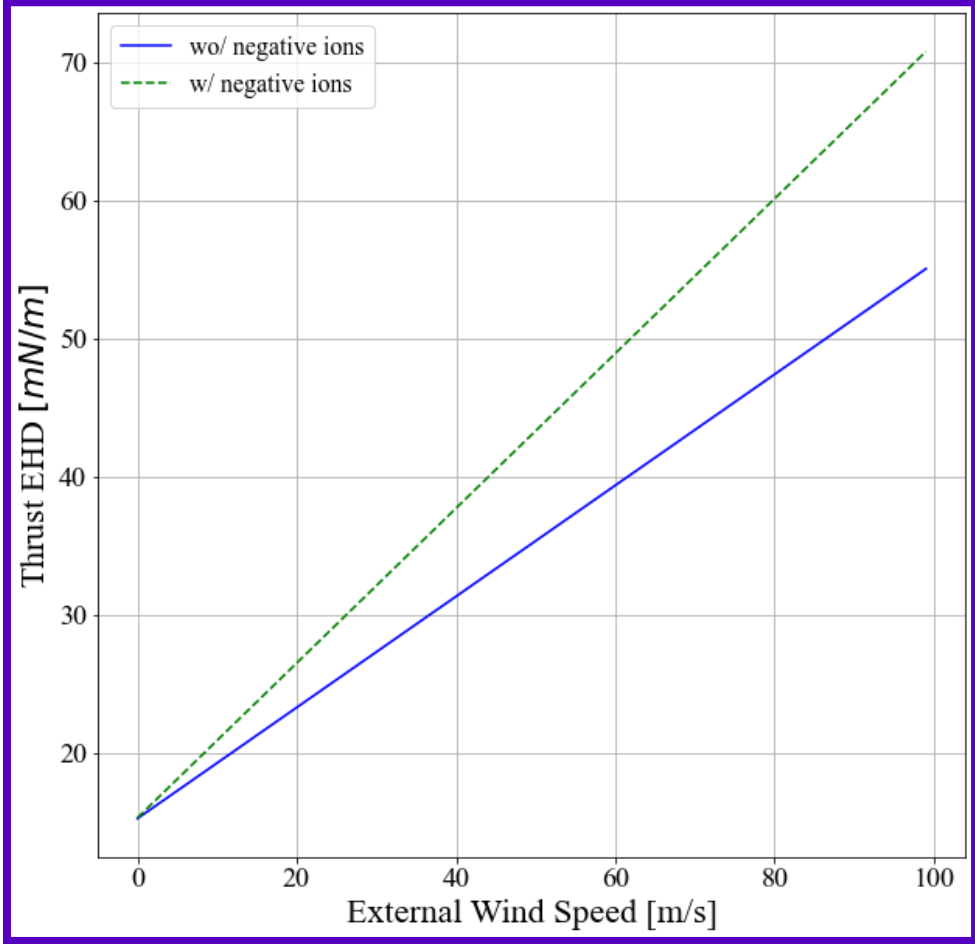
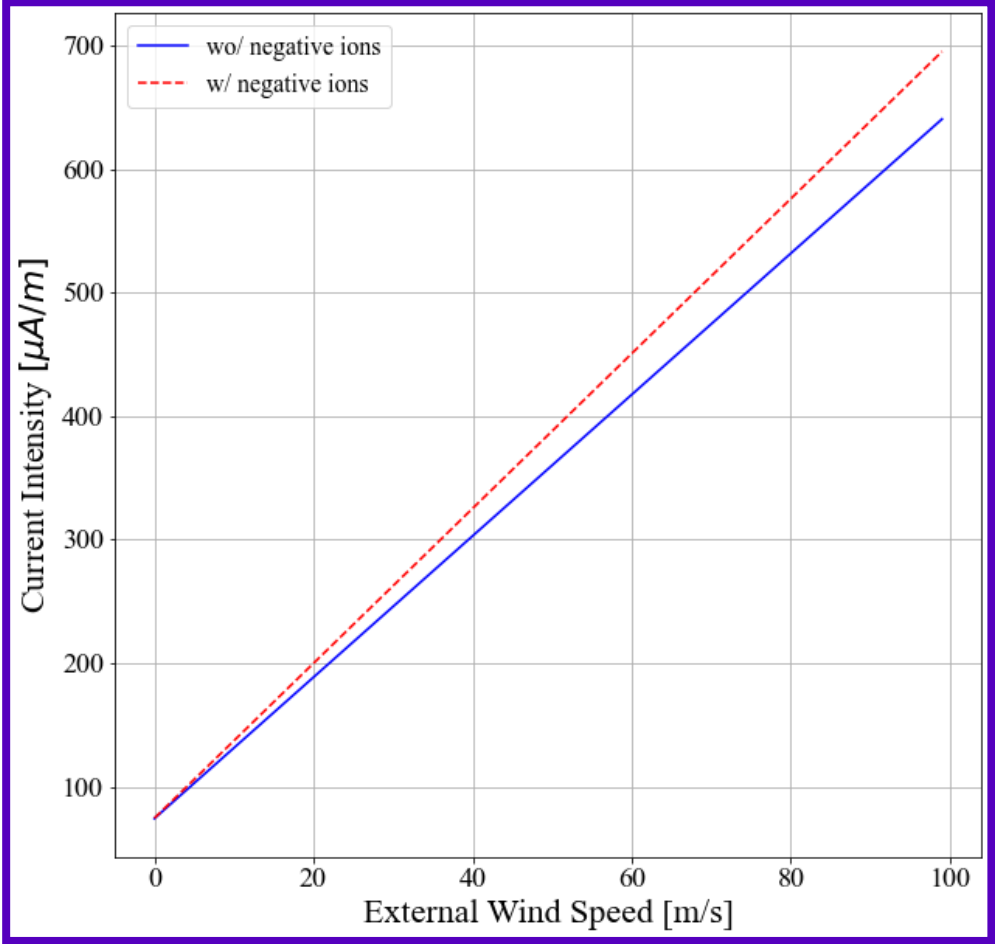
$\Rightarrow$  Charge **generation** increases  
 $\Rightarrow$  Depends only on  $||\mathbf{E}||$  :  $\alpha = \alpha(||\mathbf{E}||)$

$\left. \begin{array}{l} \Rightarrow \text{Charge generation increases} \\ \Rightarrow \text{Depends only on } ||\mathbf{E}|| : \alpha = \alpha(||\mathbf{E}||) \end{array} \right\} \Rightarrow ||\mathbf{E}|| \text{ must increase}$

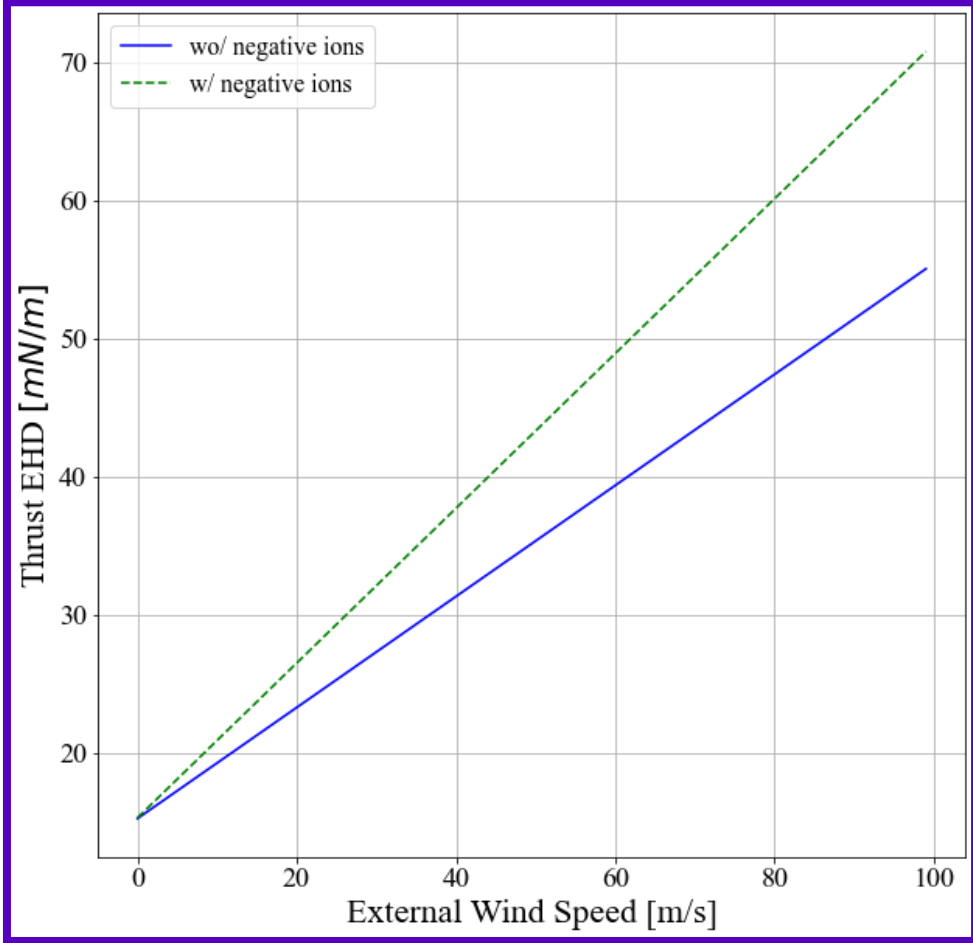
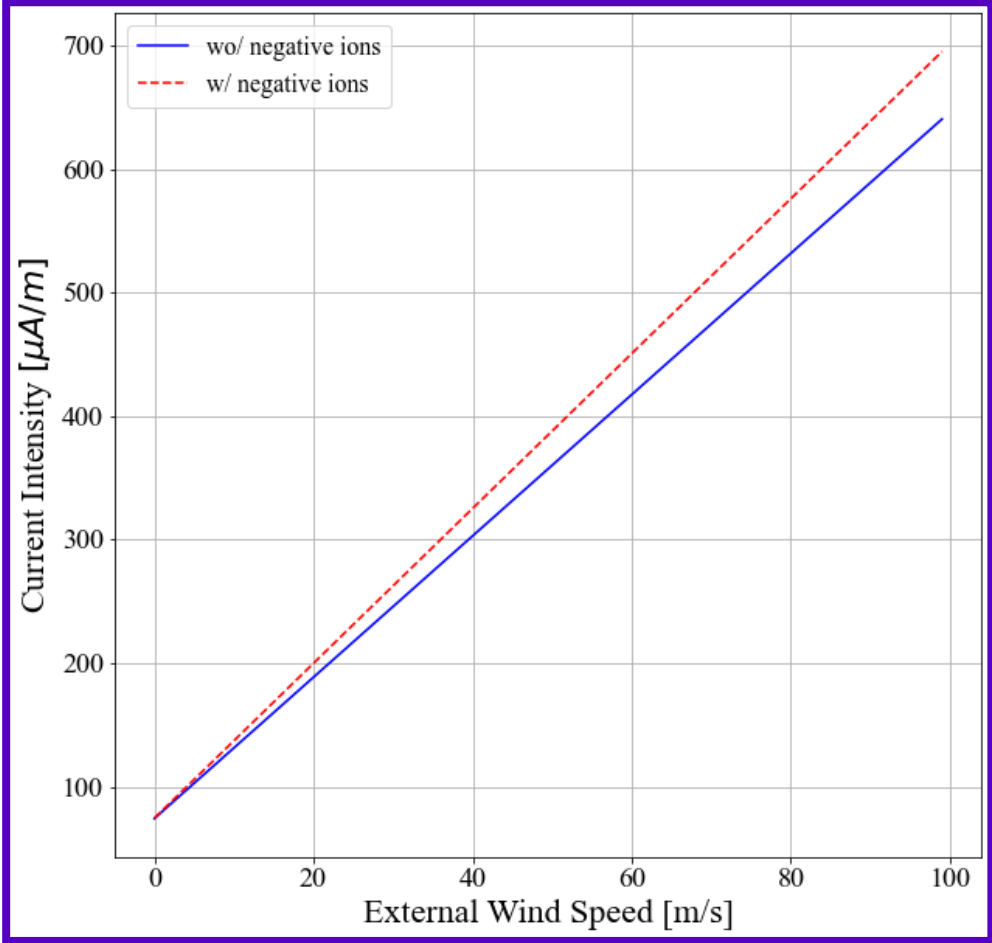
# Negative ions



# Negative ions



# Negative ions

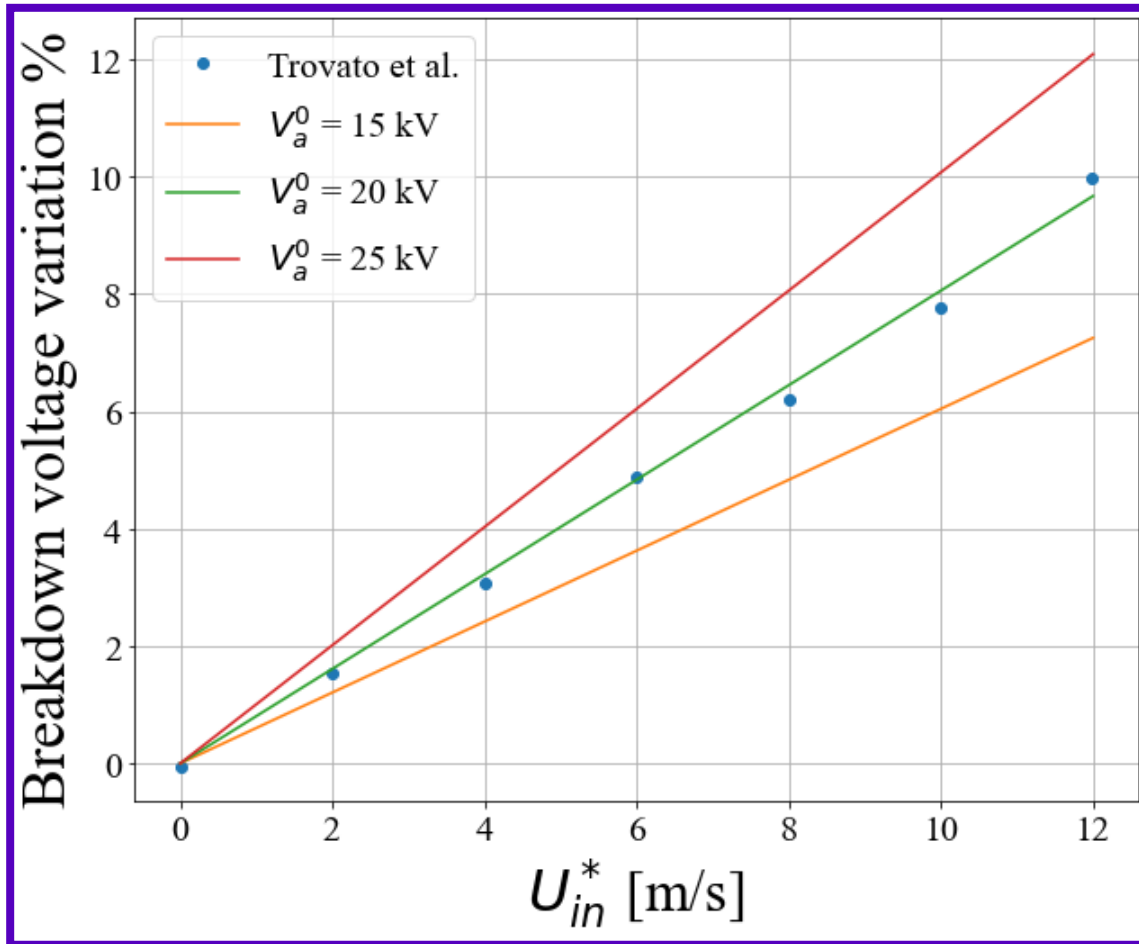


⇒ Negative ions also contribute to  $\|\mathbf{E}\|$  enhancement

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# Breakdown



$$V^1 \propto U_{in}$$

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