Two-Way Electro-Hydro-Dynamic Coupling Modeling of Ionic Wind with an External Flow

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Introduction to Electro-hydro-dynamics (EHD)

Electro-hydro-dynamic (EHD) systems rely on acceleration of a charged fluid, directly converting electrical power into mechanical power.

Chapman first modeled the behavior of intensity with an external flow :

$I = CV(V - V_i) \; + \; Ku(V - V_i)$

V_i : inception voltage
C, K : constants

Seville Chapman. Corona point current in wind. Journal of Geophysical Research, 75(12):2165–2169, April 1970.

How can EHD be modeled in the presence of an external flow ?

Dimensionless Parameters

Electro-drift Pellet number : $P_e = \mu\varphi_a / D_\rho$

EHD Reynolds number : $R_e = \varphi_a\sqrt{\varepsilon_0\rho_f}/\nu_f$

EHD electric speed : $U_e = \dfrac{\varphi_a}{r_c}\sqrt{\dfrac{\varepsilon_0}{\rho_f}}$

EHD Mach number : $M_c = \dfrac{U_e}{\mu\varphi_a/r_c}$

Speed ratio : $\beta = \dfrac{U_{in}}{U_e}$

Ionization coefficient : $\alpha = \beta_g e^{-\frac{E_i}{E}}$

Attachement coefficient : η

Photoionization : S

Two-Way Dimensionless Corona Discharge and Fluid Model

Modeling of a fully coupled corona discharge with the external fluid flow can be achieved by adding a fluid speed to the convective part of the charge conservation equations and an electing forcing to the Navier-Stokes equations. The dimensionless two-way coupled model is solved in **two regions**, one near the corona where the cold plasma is set up, and the other on the drift region.

Inner domain Ω_1

$$\begin{cases} \Delta\varphi = 0 \\ \nabla\cdot[n_p(-\nabla\varphi + M_c\mathbf{u})] = \alpha\|n_e\mathbf{E}\| \\ \nabla\cdot[n_e(\nabla\varphi + M_c\mathbf{u})] = (\alpha-\eta)\|n_e\mathbf{E}\| \\ \nabla\cdot[n_n(\nabla\varphi + M_c\mathbf{u})] = \eta\|n_e\mathbf{E}\| \end{cases}$$

Outer domain Ω_2

$$\begin{cases} \Delta\varphi = -n_p + n_n \\ \nabla\cdot[n_p(-\nabla\varphi + M_c\mathbf{u}) - \dfrac{1}{P_e}\nabla n_p] = \gamma S \\ \nabla\cdot[n_n(\nabla\varphi + M_c\mathbf{u}) - \dfrac{1}{P_e}\nabla n_n] = 0 \end{cases}$$

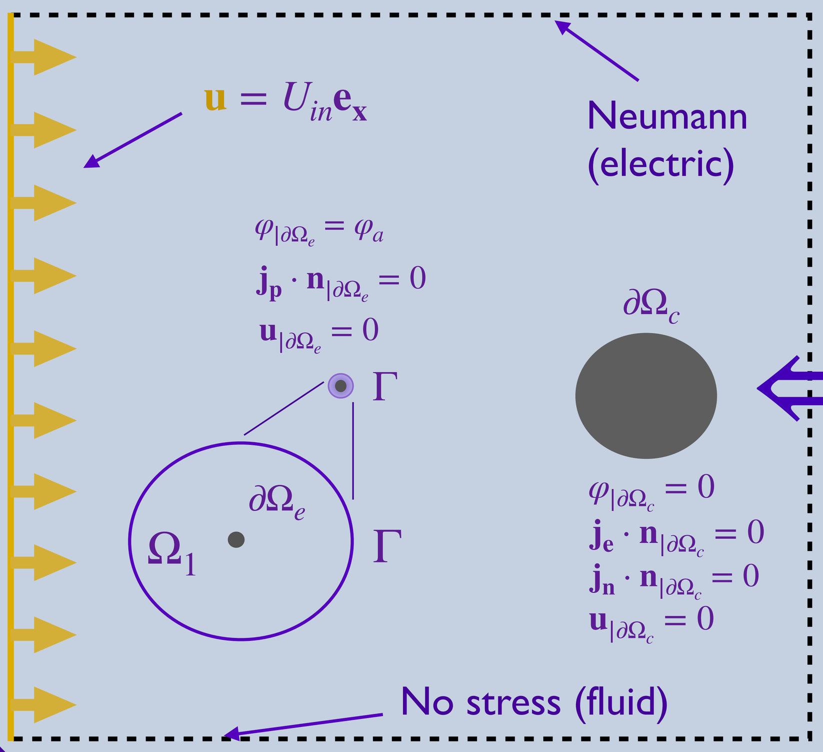
Full domain Ω

$$\begin{cases} \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \dfrac{\Delta\mathbf{u}}{R_e} - M_a(n_p - n_n)\nabla\varphi \\ \nabla\cdot\mathbf{u} = 0 \end{cases}$$

Asymptotic expiation with respect to the EHD Mach Number : $M_c = \dfrac{U_e}{\mu\varphi_a/r_c} \approx 10^{-2}$

Nicolas Monrolin and Franck Plouraboué. Journal of Computational Physics, 443:110517, October 2021.
Francesco Picella, David Fabre, and Franck Plouraboué. AIAA Journal, pages 1–12, April 2024.

Typical Configurations and Boundary Conditions



Asymptotic Expansion of the Two-Way Two-Domain Model

The 0th-order expansion was solved by Monrolin et al. and Picella et al.

Inner domain Ω_1

$\Delta\varphi^0 = 0$

$\nabla\cdot[-n_p^0\nabla\varphi^0] = \alpha^0 n_e^0|\mathbf{E}^0|$

$\nabla\cdot[n_e^0\nabla\varphi^0] = (\alpha^0-\eta^0)n_e^0|\mathbf{E}^0|$

$\nabla\cdot[n_n^0\nabla\varphi^0] = \eta^0 n_e^0|\mathbf{E}^0|$

Outer domain Ω_2

$\Delta\varphi^0 = -n_p^0 + n_n^0$

$\nabla\cdot[-n_p^0\nabla\varphi^0 - \dfrac{1}{P_e}\nabla n_p^0] = \gamma S^0$

$\nabla\cdot[-n_n^0\nabla\varphi^0 - \dfrac{1}{P_e}\nabla n_n^0] = 0$

Full domain Ω

$\mathbf{u}^0\cdot\nabla\mathbf{u}^0 = -\nabla p^0 + \dfrac{1}{R_e}\Delta\mathbf{u}^0 - M_a(n_p^0 - n_n^0)\nabla\varphi^0$

$\nabla\cdot\mathbf{u}^0 = 0$

At the 1st order the linear problem receives a source term from the 0th order problem :

Inner domain Ω_1

$\Delta\varphi^1 = 0$

$\nabla\cdot[-n_p^1\nabla\varphi^0 - n_p^0\nabla\varphi^1] - \alpha^1 n_e^0|\mathbf{E}^0| - \alpha^0 n_e^1|\mathbf{E}^1| - \alpha^0 n_e^0|\mathbf{E}^1| = -\nabla\cdot(n_p^0\mathbf{u}^0)$

$\nabla\cdot[-n_e^1\nabla\varphi^0 - n_e^0\nabla\varphi^1] + (\alpha^1-\eta^1)n_e^0|\mathbf{E}^0| + (\alpha^0-\eta^0)n_e^1|\mathbf{E}^0| + (\alpha^0-\eta^0)n_e^0|\mathbf{E}^1| = 0$

$\nabla\cdot[-n_n^1\nabla\varphi^0 - n_n^0\nabla\varphi^1] + \eta^1 n_e^0|\mathbf{E}^0| + \eta^0 n_e^1|\mathbf{E}^0| + \eta^0 n_e^0|\mathbf{E}^1| = 0$

Outer domain Ω_2

$\Delta\varphi^1 = -n_p^1$

$\nabla\cdot[-n_p^1\nabla\varphi^0 - n_p^0\nabla\varphi^1 - \dfrac{1}{P_e}\nabla n_p^0] = \gamma S^1 - \nabla\cdot(n_p^0\mathbf{u}^0)$

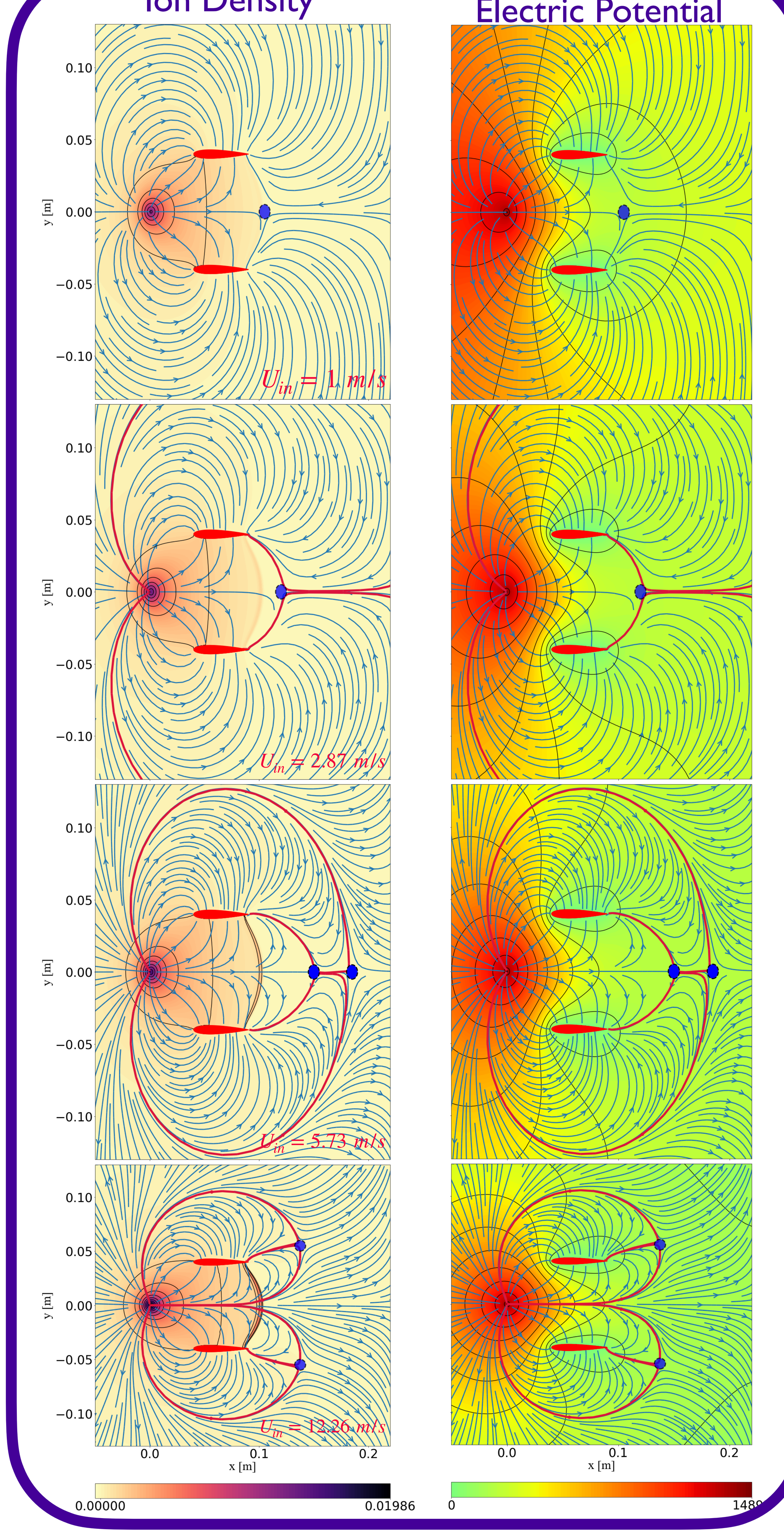
$\nabla\cdot[-n_n^1\nabla\varphi^0 - n_n^0\nabla\varphi^1 - \dfrac{1}{P_e}\nabla n_n^0] = -\nabla\cdot(n_n^0\mathbf{u}^0)$

Full domain Ω

$\mathbf{u}^1\cdot\nabla\mathbf{u}^0 + \mathbf{u}^0\cdot\nabla\mathbf{u}^1 = -\nabla p^1 + \dfrac{1}{R_e}\Delta\mathbf{u}^1 - M_a n_p^1\nabla\varphi^0 - M_a n_p^0\nabla\varphi^1$

$\nabla\cdot\mathbf{u}^1 = 0$

Electric Field Lines

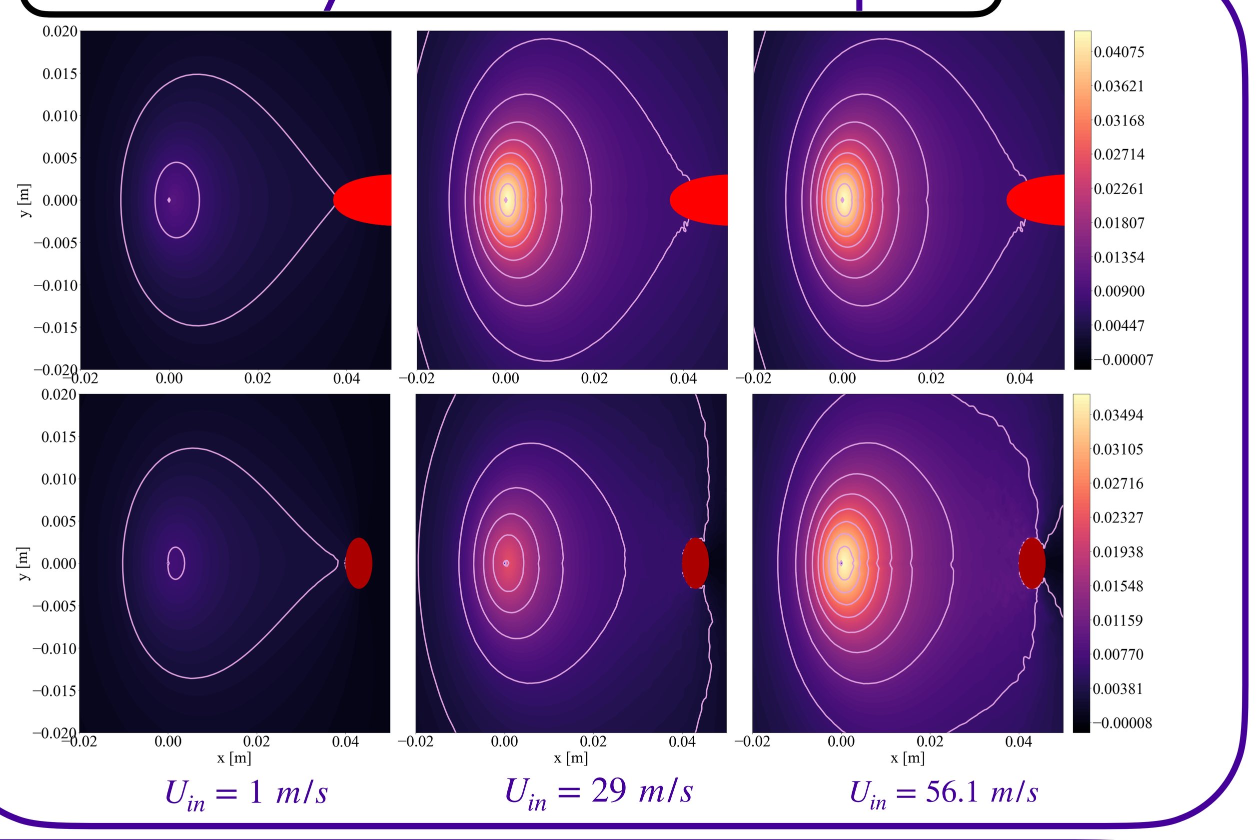


Solver structure

The asymptotic expansion leads to **two one-way** problems nonlinear at the 0th order and linear at the 1st order solved in the following order :

Electric 0th-order
⇓
Fluid 0th-order
⇓
Electric 1st-order
⇓
Fluid 1st-order

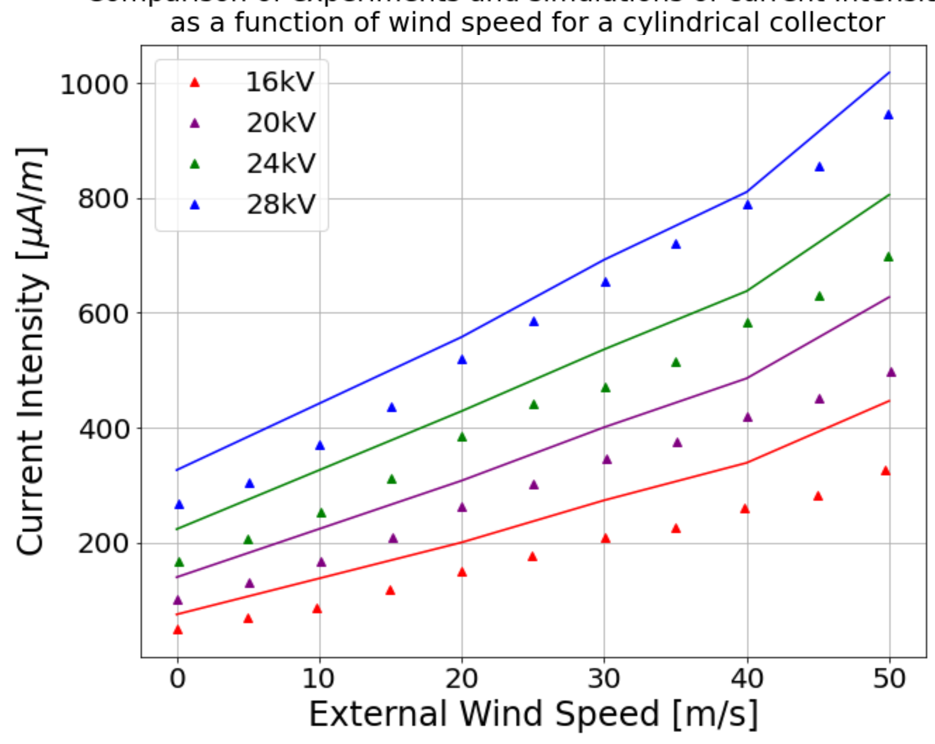
Ion Density Fields with Flow Speed



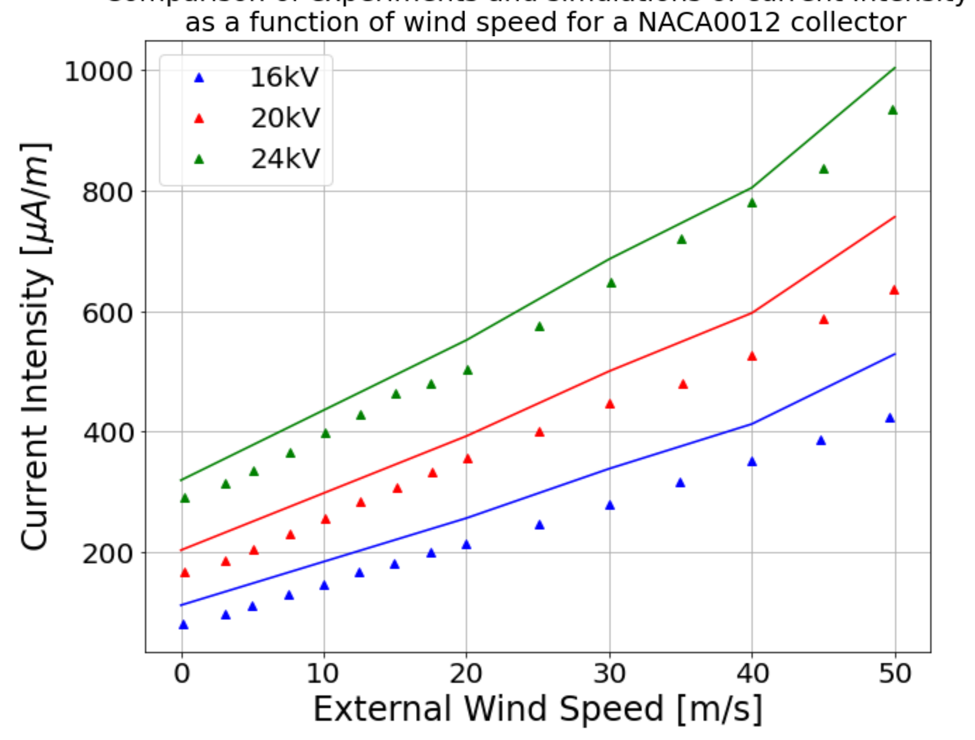
Comparison with Experiments

Current intensity : $I = \int_{\partial\Omega_e}\mu(n_p - n_e - n_n)\nabla\varphi = -\int_{\partial\Omega_c}\mu(n_p - n_e - n_n)\nabla\varphi$

Comparison of experiments and simulations of current Intensity as a function of wind speed for a cylindrical collector



Comparison of experiments and simulations of current Intensity as a function of wind speed for a NACA0012 collector



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