Two-Way Electro-Hydro-Dynamic Coupling Modeling of Ionic Wind with an External Flow

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Introduction to Electro-hydro-dynamics (EHD)

Electro-hydro-dynamic (EHD) systems rely on acceleration of a charged fluid, directly converting electrical power into mechanical power.

Chapman first modeled the behavior of intensity with an external flow :

$I = CV(V - V_i) \; + \; Ku(V - V_i)$ $\quad$ V_i : inception voltage, C, K : constants

Seville Chapman. Corona point current in wind. Journal of Geophysical Research, 75(12):2165–2169, April 1970.

How can EHD be modeled in the presence of an external flow ?

Two-Way Corona Discharge and Fluid Model

Modeling of a fully coupled corona discharge with the external fluid flow can be achieved by adding a fluid speed to the convective part of the charge conservation equations and an electing forcing to the Navier-Stokes equations :

Electrostatic Poisson $\quad \Delta^*\varphi^* = -\dfrac{e}{\varepsilon_0}(n_p^* - n_n^* - n_e^*)$

Electron Conservation $\quad \nabla^* \cdot \underbrace{[n_e^*(-\mu_e \nabla^*\varphi^* + \mathbf{u}^*) - D_\rho \nabla^* n_e^*]}_{\mathbf{j_e}^*} = (\alpha^* - \eta)\|\mathbf{j_e^*}\| + S^*$

Positive Charge Conservation $\quad \nabla \cdot [n_p^*(-\mu_e \nabla^*\varphi^* + \mathbf{u}^*) - D_\rho \nabla^* n_p^*] = \alpha^*\|\mathbf{j_e^*}\| + S^*$

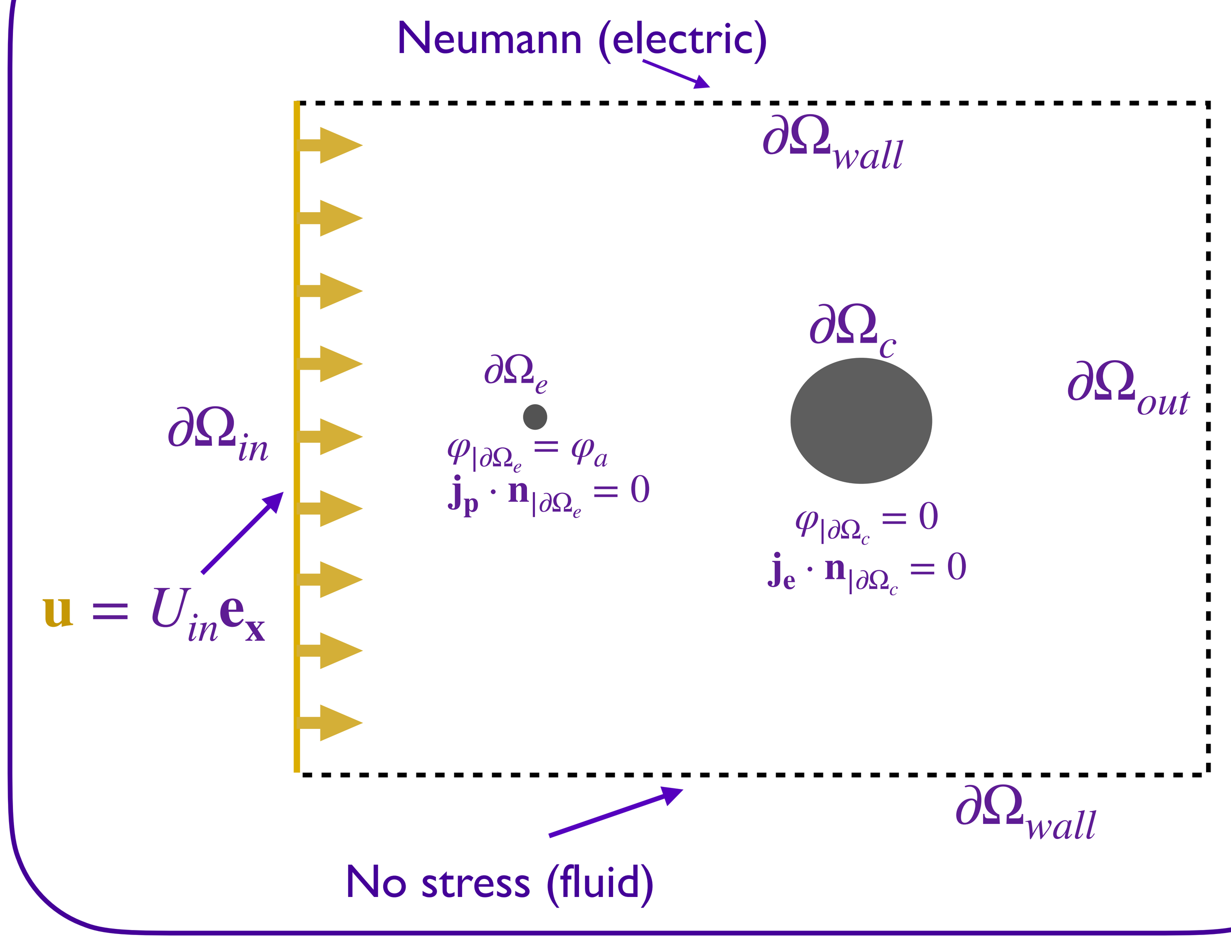
Negative Charge Conservation $\quad \nabla^* \cdot [n_n^*(-\mu_n \nabla^*\varphi^* + \mathbf{u}^*) - D_\rho \nabla^* n_n^*] = \eta^*\|\mathbf{j_e^*}\|$

Stationary Navier-Stokes $\quad \rho_f \mathbf{u}^* \cdot \nabla^* \mathbf{u}^* = -\nabla^* p^* + \mu_f \Delta^* \mathbf{u}^* - (n_p^* - n_e^* - n_n^*)\nabla\varphi^*$, $\quad \nabla^* \cdot \mathbf{u}^* = 0$

$\alpha = \beta_g e^{-\frac{E_i}{E}}$: Ionisation coefficient $\qquad$ η : Attachement coefficient $\qquad$ S : Photoionisation

Nicolas Monrolin and Franck Plouraboué. Multi-scale two-domain numerical modeling of stationary positive DC corona discharge/drift-region coupling. Journal of Computational Physics, 443:110517, October 2021.
Francesco Picella, David Fabre, and Franck Plouraboué. Numerical Simulations of Ionic Wind Induced by Positive DC-Corona Discharges. AIAA Journal, pages 1–12, April 2024.

Typical Configurations and Boundary Conditions



Dimensionless Parameters

Electro-drift Pellet number : $P_e = \mu\varphi_a / D_\rho$

EHD Reynolds number : $R_e = \varphi_a \sqrt{\varepsilon_0 \rho_f} / \nu_f$

EHD electric speed : $U_e = \dfrac{\varphi_a}{r_c}\sqrt{\dfrac{\varepsilon_0}{\rho_f}}$

EHD Mach number : $M_c = \dfrac{U_e}{\mu\varphi_a / r_c}$

Speed ratio : $\beta = \dfrac{Ue}{U_{in}}$

Two-Way Two Domain Dimensionless Model

The dimensionless two-way coupled model is solved in **two regions**, one near the corona where the cold plasma is set up, and the other on the drift region.

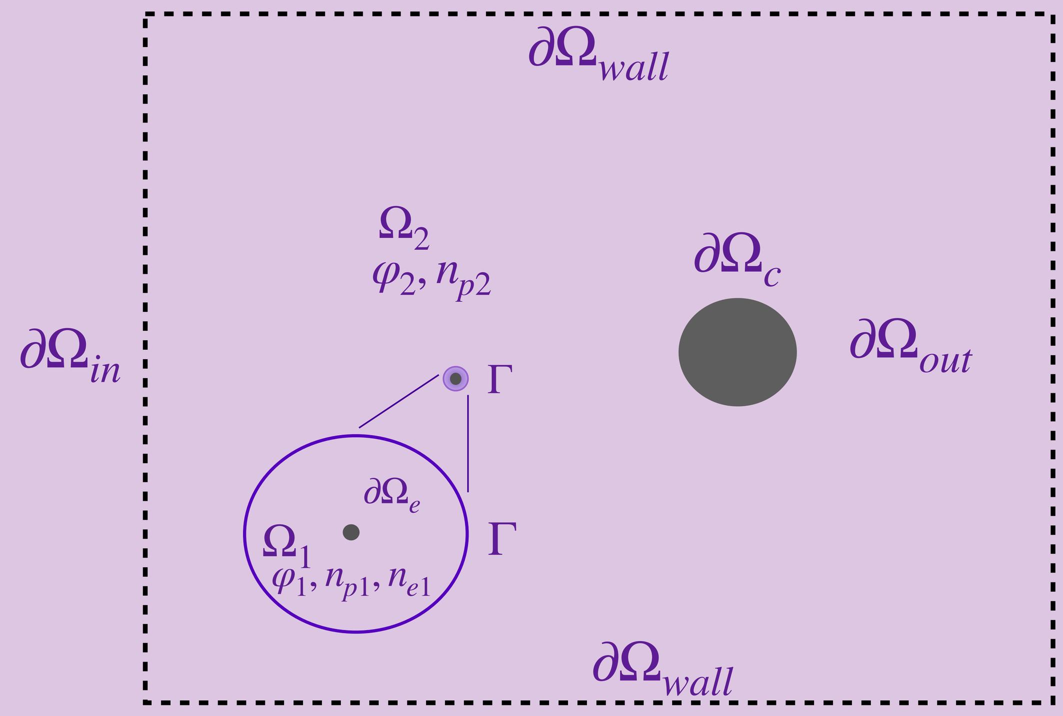
Inner domain Ω_1 $\quad \begin{cases} \Delta\varphi = 0 \\ \nabla \cdot [n_p(-\nabla\varphi + M_c\mathbf{u})] = \alpha\|n_e\mathbf{E}\| \\ \nabla \cdot [n_e(-\nabla\varphi + M_c\mathbf{u})] = (\alpha - \eta)\|n_e\mathbf{E}\| \end{cases}$

Outer domain Ω_2 $\quad \begin{cases} \Delta\varphi = -n_p \\ \nabla \cdot [n_p(-\nabla\varphi + M_c\mathbf{u}) - \dfrac{1}{P_e}\nabla n_p] = \gamma S \end{cases}$

Full domain Ω $\quad \begin{cases} \mathbf{u} \cdot \nabla\mathbf{u} = -\nabla p + \dfrac{1}{R_e}\Delta\mathbf{u} - M_a n_p \nabla\varphi \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$

Asymptotic expiation with respect to the EHD Mach Number : $M_c = \dfrac{U_e}{\mu\varphi_a / r_c} \approx 10^{-2}$

$\Rightarrow \mathbf{X} = \mathbf{X}^0 \; + \; M_c \, \mathbf{X}^1 + O(M_c^2)$



Asymptotic Expansion of the Two-Way Two-Domain Model

The asymptotic expansion leads to **two one-way** problems nonlinear at the 0th order and linear at the 1st order solved in the following order :

Electric 0th-order
⇓
Fluid 0th-order
⇓
Electric 1st-order
⇓
Fluid 1st-order

The 0th-order expansion was solved by Monrolin et al. and Picella et al.

Inner domain Ω_1

$\Delta\varphi^0 = 0$

$\nabla \cdot [-n_p^0 \nabla\varphi^0] = \alpha^0 n_e^0 |\mathbf{E^0}|$

$\nabla \cdot [-n_e^0 \nabla\varphi^0] = (\alpha^0 - \eta^0) n_e^0 |\mathbf{E^0}|$

Outer domain Ω_2

$\Delta\varphi^0 = -n_p^0$

$\nabla \cdot [-n_p^0 \nabla\varphi^0 - \dfrac{1}{P_e}\nabla n_p^0] = \gamma S^0$

Full domain Ω

$\mathbf{u}^0 \cdot \nabla\mathbf{u}^0 = -\nabla p^0 + \dfrac{1}{R_e}\Delta\mathbf{u}^0 - M_a n_p^0 \nabla\varphi^0$

$\nabla \cdot \mathbf{u}^0 = 0$

At the 1st order the linear problem receives a source term from the 0th order problem :

Inner domain Ω_1

$\Delta\varphi^1 = 0$

$\nabla \cdot [-n_p^1 \nabla\varphi^0 - n_p^0 \nabla\varphi^1] - \alpha^1 n_e^0 |\mathbf{E^0}| - \alpha^0 n_e^1 |\mathbf{E^1}| - \alpha^0 n_e^0 |\mathbf{E^1}| = -\nabla \cdot (n_p^0 \mathbf{u^0})$

$\nabla \cdot [-n_e^1 \nabla\varphi^0 - n_e^0 \nabla\varphi^1] + (\alpha^1 - \eta^1) n_e^0 |\mathbf{E^0}| + (\alpha^0 - \eta^0) n_e^1 |\mathbf{E^0}| + (\alpha^0 - \eta^0) n_e^0 |\mathbf{E^1}| = 0$

Outer domain Ω_2

$\Delta\varphi^1 = -n_p^1$

$\nabla \cdot [-n_p^1 \nabla\varphi^0 - n_p^0 \nabla\varphi^1 - \dfrac{1}{P_e}\nabla n_p^0] = \gamma S^1 - \nabla \cdot (n_p^0 \mathbf{u^0})$

Full domain Ω

$\mathbf{u}^1 \cdot \nabla\mathbf{u}^0 + \mathbf{u}^0 \cdot \nabla\mathbf{u}^1 = -\nabla p^1 + \dfrac{1}{R_e}\Delta\mathbf{u}^1 - M_a n_p^1 \nabla\varphi^0 - M_a n_p^0 \nabla\varphi^1$

$\nabla \cdot \mathbf{u}^1 = 0$

Numerical Methods

Solving the 0th-order fluid problem with **P2-P2 stabilized finite elements** and linearizing it with a Newton-Ralphson iteration using **FreeFem++** and **Stabfem.**

$$L_{NS}(\mathbf{u}; \mathbf{u}, p) := -1/R_e \Delta\mathbf{u} + \mathbf{u} \cdot \nabla\mathbf{u} + \nabla p = -M_a n_p \nabla\varphi$$

$$\Rightarrow \int_\Omega \frac{1}{R_e}\nabla\mathbf{u}_h \cdot \nabla\mathbf{v}_h + (\mathbf{u}_h \cdot \nabla)\mathbf{u}_h \cdot \mathbf{v}_h - p_h \nabla \cdot \mathbf{v}_h - q_h \nabla \cdot \mathbf{u}_h \; + M_a n_p \nabla\varphi \mathbf{v}_h \, d\Omega \; + \; \text{BC}$$

SUPG/PSPG $\quad + \displaystyle\sum_{t \in T_h} \int_{T_h} \tau_{SUPG} L_{NS}(\mathbf{u}_h; \mathbf{u}_h, p_h) \cdot ((\mathbf{u}_h \cdot \nabla)\mathbf{v}_h + \nabla q_h))$

Grad-div $\quad + \displaystyle\sum_{t \in T_h} \int_{T_h} \gamma_{grad-div}(\nabla \cdot \mathbf{u}_h)(\nabla \cdot \mathbf{v}_h)$

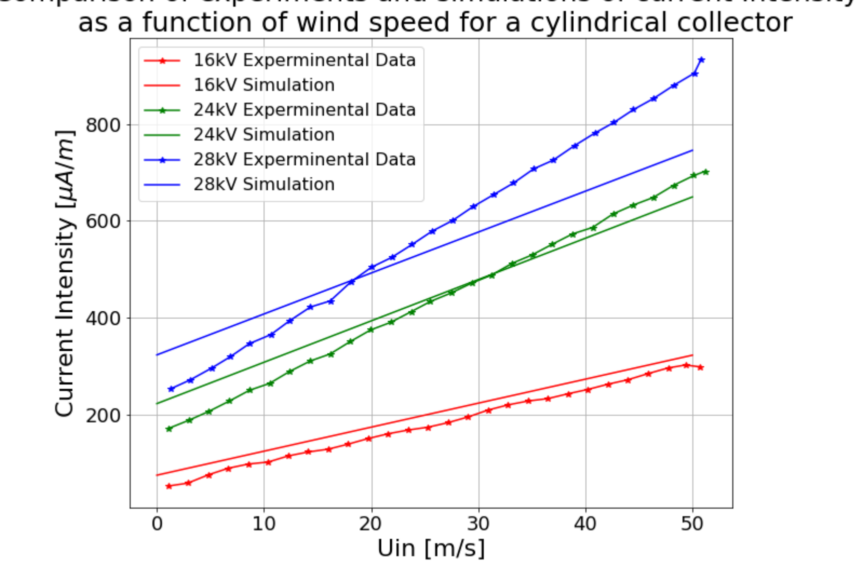
SUPG/PSPG $\quad - \displaystyle\sum_{t \in T_h} \int_{T_h} \tau_{SUPG} n_{p,h}^0 \nabla\varphi_h^0((\mathbf{u}_h^0 \cdot \nabla)\mathbf{v}_h + \nabla q_h)$

Hecht, F. (2012). New development in FreeFem++. Journal of numerical mathematics, 20(3-4), 251-266.
M. Braack, E. Burman, V. John, G. Lube, Computer Methods in Applied Mechanics and Engineering, 2007, https://doi.org/10.1016/j.cma.2006.07.011.
Ern and J.-L. Guermond, Texts in Applied Mathematics, Springer, 2021, https://doi.org/10.1007/978-3-030-56341-7

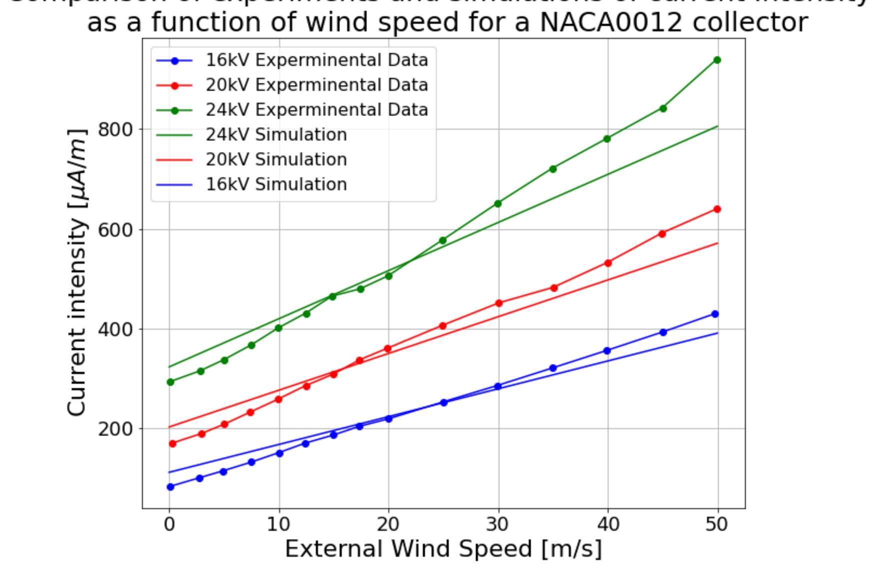
Comparison with Experiments

Current intensity : $\quad I = \displaystyle\int_{\partial\Omega_e} \mu(n_p - n_e - n_n)\nabla\varphi = -\int_{\partial\Omega_c} \mu(n_p - n_e - n_n)\nabla\varphi$

Comparison of experiments and simulations of current Intensity as a function of wind speed for a cylindrical collector



Comparison of experiments and simulations of current Intensity as a function of wind speed for a NACA0012 collector



C. Guerra-Garcia, N. C. Nguyen, T. Mouratidis, and M. Martinez-Sanchez. Corona Discharge in Wind for Electrically Isolated Electrodes. Journal of Geophysical Research: Atmospheres, 125(16):e2020JD032908, August 2020.
Sylvain Grosse, Nicolas Benard, Eric Moreau, Electroaerodynamic thrusters: Influence of a freestream on the current, ionic wind, and force produced by a DC corona discharge, Journal of Electrostatics